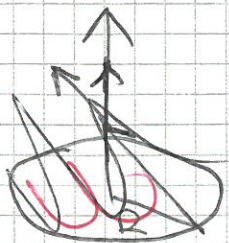
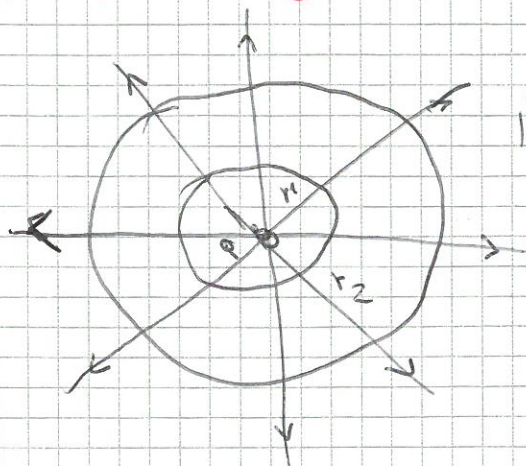


~~PROBLEMA~~



SUPERFICIE GAUSSIANA

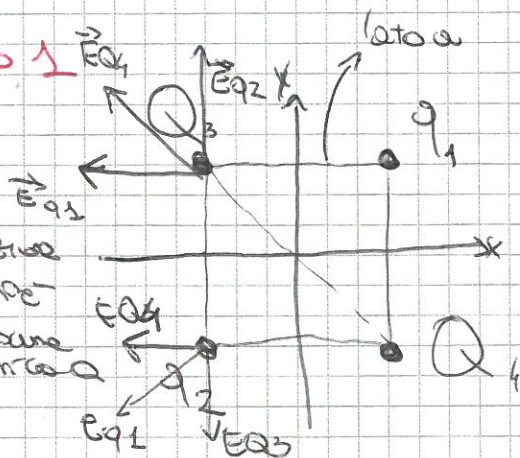


Il flusso è uguale in tutte e due le superfici

ESERCIZIO 1

Le cariche
 $Q = q$

a) Trovare relazione
tra Q e q affinché
la forza risultante
parallela all'asse x
sia nulla



$q = q_1 = q_2$ Tutte positive
 $Q = Q_3 = Q_4$

$$\vec{E}_{q_1}(Q_1) = \frac{1}{4\pi\epsilon_0} \frac{q}{a^2} (-\hat{x}) \quad \vec{E}_{q_2}(Q_1) = \frac{1}{4\pi\epsilon_0} \frac{q}{a^2} (\hat{y}) \quad \vec{E}_{Q_4}(Q_1) = \frac{1}{4\pi\epsilon_0} \frac{Q}{\frac{a^2}{2}} \left[\cos 45^\circ (-\hat{x}) + \sin 45^\circ (\hat{y}) \right]$$

$$\vec{F} = Q \vec{E}$$

$$\vec{E}_{\text{Tot}}(Q_1) = \frac{1}{4\pi\epsilon_0} \frac{1}{a^2} \left(q + \frac{Q}{\sqrt{2}} \right) (-\hat{x}) + \frac{1}{4\pi\epsilon_0} \frac{1}{a^2} \left(q + \frac{Q}{\sqrt{2}} \right) \hat{y}$$

$E_x = E_y$

$$\vec{F}_{\text{Tot}} = Q \vec{E}_{\text{Tot}} = 0$$

$$\vec{E}_{\text{Tot}} = \sqrt{E_x^2 + E_y^2} = \sqrt{2} E_x = \sqrt{2} E_y$$

Quindi affinché $\vec{F} = Q \vec{E} = 0 \rightarrow E_x = 0 \rightarrow$ unica soluzione possibile

$$E_x = q + \frac{Q}{\sqrt{2}} = 0 \rightarrow q = -\frac{Q}{\sqrt{2}}$$

b) Data tale relazione determinare

la forza agente sulle particelle di carica q .

$$q + \frac{Q}{\sqrt{2}} = 0$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{Q}{a^2} (-\hat{x}) \quad \vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{Q}{a^2} (-\hat{y})$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{Q}{a^2} \left(\frac{1}{r_1} (-\hat{x}) + \frac{1}{r_2} (-\hat{y}) \right)$$

$$\vec{E}_{\text{Tot}} = \frac{1}{4\pi\epsilon_0} \frac{1}{a^2} \left(Q + \frac{Q}{2\sqrt{2}} \right) (-\hat{x}) + \frac{1}{4\pi\epsilon_0} \frac{1}{a^2} \left(Q + \frac{Q}{2\sqrt{2}} \right) (-\hat{y}) = \frac{1}{4\pi\epsilon_0} \frac{1}{a^2} \left(\frac{7Q}{2} \right) (-\hat{x}) + \frac{1}{4\pi\epsilon_0} \frac{1}{a^2} \left(\frac{7Q}{2} \right) (-\hat{y})$$

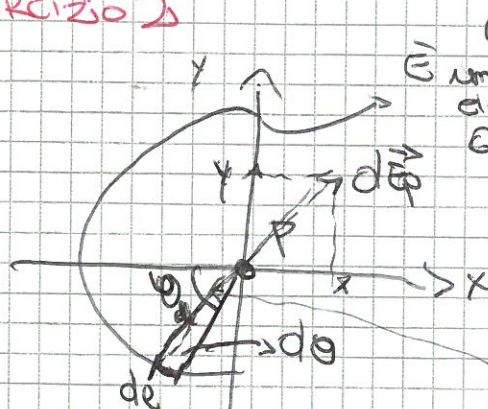
Ma noi sappiamo che

$$Q + \frac{Q}{2\sqrt{2}} = Q - \left(\frac{Q}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} \right) = Q - \frac{Q}{2} = \frac{Q}{2}$$

$$\vec{F}_{\text{Tot}} = q \vec{E}_{\text{Tot}}$$

$$E_{\text{Tot}} = \sqrt{E_x^2 + E_y^2} = \sqrt{2} E_x = \sqrt{2} E_y = \frac{\sqrt{2}}{4\pi\epsilon_0} \frac{7Q}{8a^2}$$

ESERCIZIO 2



SEMICIRCOLO
una data unif. carica. Ricavare l'intensità del campo elettrico nel punto P in funzione della carica totale Q dell'arco e del raggio a.
L'arco ha dens. lineare uniforme
 $\lambda = \left[\frac{C}{m} \right] \rightarrow Q = \lambda(n \cdot a) \rightarrow \text{Semicirco}$

Prendiamo un $d\ell$ qualunque che forma un angolo θ con x

Questo $d\ell$ è compreso ovviamente tra $[-\pi/2, \pi/2]$ a seconda di dove si prende $d\ell$

Il arco se piccolo forma un piccolo arco $\rightarrow d\ell = a d\theta$. Inoltre $dq = \lambda d\ell$

formule
di geometria
arco

$a d\theta$

Questo $d\ell$ dal punto P farà un piccolo campo elettrico $d\vec{E}_0$

da $d\ell$ la
carica su $d\ell$
che forma il campo

$$d\vec{E}_0 = \frac{1}{4\pi\epsilon_0} \frac{dq}{a^2} [\cos\theta \hat{x} + \sin\theta \hat{y}]$$

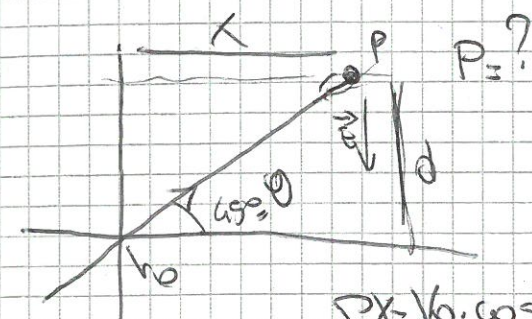
Per avere tutto il campo
basta integrare

La x e y componenti lungo y fra II e III quadrante la parte y è essendo opposte
si annulla quindi ci rimangono solo E_x

$$dE_x = \frac{1}{4\pi\epsilon_0} \frac{\lambda}{a} \cos\theta d\theta \Rightarrow \vec{E}_P = \hat{x} \int_{-\pi/2}^{\pi/2} \frac{\lambda}{4\pi\epsilon_0 a} \cos\theta d\theta = \hat{x} \frac{\lambda}{4\pi\epsilon_0 a} \int_{-\pi/2}^{\pi/2} \cos\theta d\theta$$

$\frac{1}{4} \sin \theta$
 $\frac{1}{4} \cos \theta$
 $\frac{1}{2}$
 $\frac{1}{2}$

Esercizio 3



Su y c'è $\vec{F}_c = q\vec{E} = qE(-\hat{y}) = -qE\hat{y}$
 Ma $F = ma \Rightarrow \vec{a} = -\frac{qE}{m}\hat{y}$

Legge oraria $\rightarrow$

$$\begin{cases} x = v_0 \cos 45^\circ t = \frac{\sqrt{2}}{2} v_0 t \\ y = v_0 \sin 45^\circ t - \frac{1}{2} \frac{qE}{m} t^2 \end{cases}$$

Imponiamo $y=d \rightarrow d = \frac{\sqrt{2}}{2} v_0 t - \frac{1}{2} \frac{qE}{m} t^2 \rightarrow$ È un eq. d. II grado in t (solo quello non abbiamo)

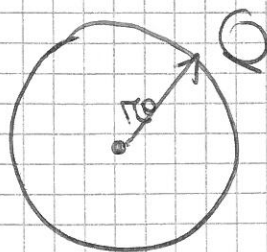
$$\frac{qEt^2}{2m} - \frac{\sqrt{2}}{2} v_0 t + 2d = 0$$

$$t_{1,2} = \frac{\sqrt{2} v_0 \pm \sqrt{2 v_0^2 - \frac{8 q E d}{m}}}{\frac{q E}{m}}$$

prendiamo solo il che
 corrisponde al moto
 poiché è $<$, equale
 vale

Una volta trovato t , lo sostituiamo nella x della legge oraria, cioè in $x = \frac{\sqrt{2}}{2} v_0 t$
 e troviamo la coordinata x di P . Y la overbmogia edera d.

ESERCIZIO 4



$E?$

Sappiamo che $\vec{E} = E(r) \hat{r}$

a) $r < r_0$

b) $r \geq r_0$

$$\Phi_S(\vec{E}) = \int \vec{E} \cdot d\vec{S} = \frac{Q_{int}}{\epsilon_0}$$

Usiamo come superficie GAUSSIANA delle SUPERFICI SFERICHE

Iniziamo con il caso a)

$$a) \quad \Phi_S(\vec{E}) = \oint_{SF(r)} (E(r) \hat{r}) \cdot d\vec{S} = \oint_{SF} E(r) \hat{r} \cdot dS \hat{r} = \oint_{SF} E(r) dS =$$

$\downarrow$
 $dS \hat{r}$

$\downarrow$
ma il campo
è lo stesso

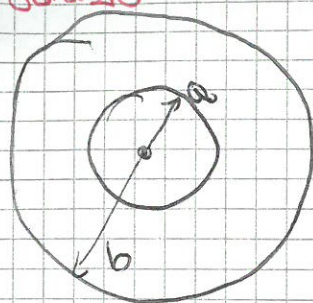
$$= E(r) \underbrace{4\pi r^2}_{\int dS} = \frac{Q_{int}}{\epsilon_0}$$

$$\text{in a } Q_{int} = 0 \rightarrow E(r) 4\pi r^2 = \frac{Q_{int}}{\epsilon_0} \rightarrow E(r) = 0$$

perché
la carica è
solo sulla
sup

$$\text{in b) } Q_{int} = Q \Rightarrow E(r) 4\pi r^2 = \frac{Q}{\epsilon_0} \rightarrow E(r) = \frac{Q}{4\pi \epsilon_0 r^2} \hat{r}$$

Esercizio 5



$$q_a = -Q$$

$$q_b = Q$$

Trova E in

$$1) r < a$$

$$2) a < r < b$$

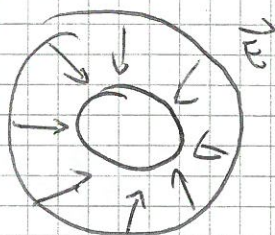
$$3) r > b$$

r : distanza tra centro e punto

Le formule e i ragionamenti sono uguali all'es 4.

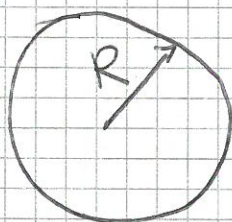
$$1) Q_{int} = 0 \quad E(r) = 0$$

$$2) Q_{int} = -Q \quad E(r) = -\frac{Q}{4\pi\epsilon_0 r^2} \hat{r}$$



$$3) Q_{int} = -Q + Q = 0 \quad E(r) = 0$$

Esercizio 6



$\rightarrow$ densità di carica

$$P = P_0 \pi$$

Il punto può essere sia fuori che dentro

Allora abbiamo due casi

$$1) r < R$$

$$2) r \geq R$$

Vale ancora la formula dei due es. precedenti

$$E(r) 4\pi r^2 = \frac{Q_{int}}{\epsilon_0}$$

Iniziamo con il caso 2)

$$2) \text{ Dovremo trovare } Q_{int} = Q = \int_{SF(R)} P(r) dV \quad \text{Volume} = \int_{SF(R)} P_0 \pi dV = \int_{SF(R)} P_0 \pi dx dy dz \rightarrow$$

$$= \int_0^R \rho_0 4\pi r^2 dr = 4\pi \rho_0 \int_0^R r^3 dr = 4\pi \rho_0 \left[\frac{r^4}{4} \right]_0^R = \pi \rho_0 R^4$$

Controllo

~~Verifica~~

Quindi, ancora nel caso 2) si trova

$$E(r) 4\pi r^2 = \frac{\pi \rho_0 R^4}{\epsilon_0} \rightarrow \vec{E}(r) = \frac{\rho_0 R^4}{4\epsilon_0 r^2} \hat{r}$$

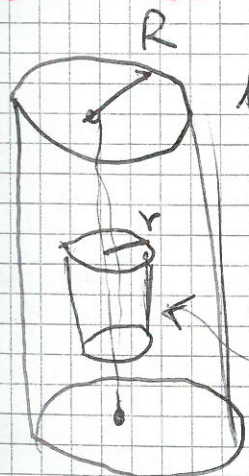
1) Ora imi 2) si trova (con il caso 1)

$$Q_{int} = \int_{S_{EL}} \rho(r) dV = \int_0^r \rho_0 r' dV = \int_0^r \rho_0 r' 4\pi r'^2 dr' = \rho_0 4\pi \int_0^r r'^3 dr' =$$

$$= 4\pi \rho_0 \left[\frac{r'^4}{4} \right]_0^r = \pi \rho_0 r^4$$

$$E(r) 4\pi r^2 = \frac{\pi \rho_0 r^4}{\epsilon_0} \rightarrow \vec{E}(r) = \frac{\rho_0 r^2}{4\epsilon_0} \hat{r}$$

Esercizio 7



1) $r < R$

2) $r \geq R$

Sappiamo che $\vec{E}(r) = E(r) \hat{r}$

Prendiamo come SUR GAUSS, un cilindro

Calcoliamo il flusso sul piccolo cilindretto

$$\Phi_0(\vec{E}) = \oint_{S_{EL}} \vec{E} \cdot d\vec{S} = \int_{S_{EL}} E(r) \hat{r} \cdot d\vec{S} = \int_{S_{EL}} E(r) \hat{r} \cdot dS \hat{r} = \int_{S_{EL}} E(r) dS$$

$$= E(r) \int_{S_{EL}} dS = E(r) \underbrace{2\pi r h}_{S_{EL}}$$

Questo risultato però vale sia che $r < R$ che $r \geq R$.

Ora:

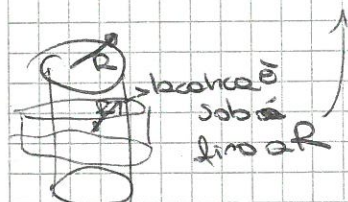
Nel caso 1

$$1) Q_{int} = \rho \pi r^2 h$$

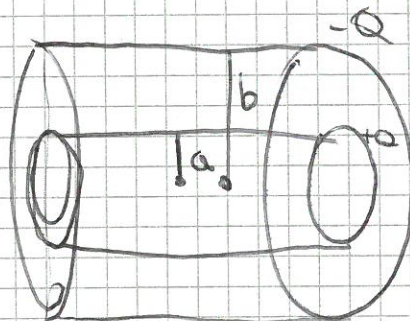
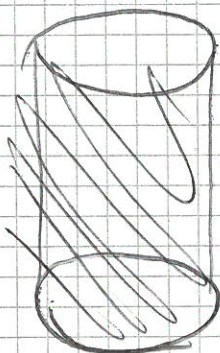
$$E(r) 2\pi r h = \frac{\rho \pi r^2 h}{\epsilon_0} \Rightarrow E(r) = \frac{\rho \pi r}{2\epsilon_0}$$

$$2) Q_{int} = \rho \pi R^2 h$$

$$E(r) 2\pi r h = \frac{\rho \pi R^2 h}{\epsilon_0} \Rightarrow E(r) = \frac{\rho R}{2\epsilon_0}$$



ESERCIZIO 8



dens. lin. d.

$$Q = d \cdot h \Rightarrow \text{hett. cilindro}$$

Prendiamo come sup. Gaussiana un cilindro di raggio r e altezza h

Distinguiamo i tre casi:

- 1) $a \leq r \leq b$
- 2) $r \geq b$

$$\Phi_S(E) = \oint_{\text{cilindro}} \vec{E} \cdot d\vec{S} = \oint_{\text{cilindro}} (E(r) \hat{r}) \cdot d\vec{S} = \int_{\text{superf.}} E(r) \hat{r} \cdot d\vec{S} = \int_{\text{superf.}} E(r) dS$$

$$= E(r) 2\pi r h = \frac{Q_{int}}{\epsilon_0}$$

Calcoliamo Q_{int}

1) In questo caso $Q_{int} = 0$

$$E(r) = 0$$

2) In questo caso $Q_{int} \neq 0$

$$da -Q + Q = 0 \quad (dh - dh = 0) \quad E(r) = 0$$

$$Q_{int} = \frac{d \cdot h}{\epsilon_0}$$

$$E(r) = \frac{d}{2\pi\epsilon_0 r}$$

ESERCIZIO 9

E' sfera carica

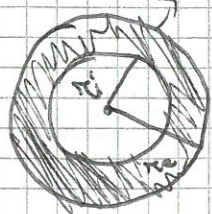
Carica totale = Q

Determinare il campo elettr.

(1) $r > r_e$

(2) $r_e > r > r_i$

(3) $r < r_i$



PRENDO SUPERGAUSS. SFERICA

$$\Phi_S(E) = \oint (\vec{E}(r) \cdot \hat{n}) \cdot dS \hat{n} = E(r) 4\pi r^2 = \frac{Q_{int}}{\epsilon_0}$$

(3) $Q_{int} = 0 \Rightarrow E(r) = 0$

(1) $Q_{int} = Q \Rightarrow \vec{E}(r) = \frac{Q}{4\pi r^2 \epsilon_0} \hat{r}$

(2) La carica totale Q sarà tipo $Q = \rho \frac{4}{3} \pi (\pi_3^3 - \pi_1^3)$

$$Q_{tot} = \rho \frac{4}{3} \pi (\pi_3^3 - \pi_1^3)$$

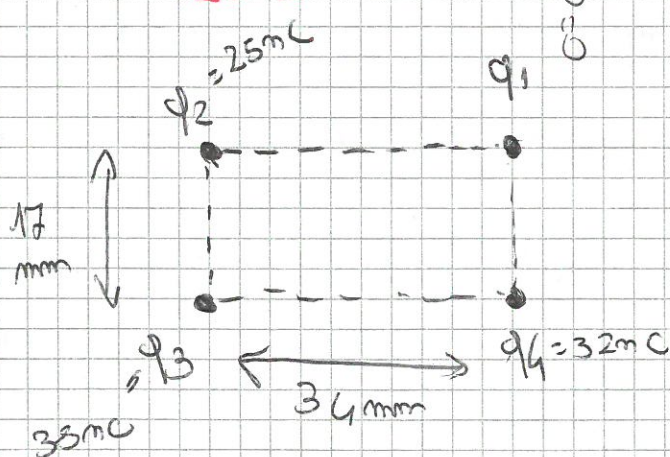
$$Q_{tot} = Q \left[\frac{\pi_3^3 - \pi_1^3}{\pi_3^3 - \pi_1^3} \right] \Rightarrow E(r) = \frac{Q}{4\pi \epsilon_0 r^2} \left[\frac{\pi_3^3 - \pi_1^3}{\pi_3^3 - \pi_1^3} \right] \frac{1}{r}$$

$$Q_{tot} = \rho \frac{4}{3} \pi \pi_3^3$$

$$\frac{Q_{tot}}{Q_2} = \frac{\rho \frac{4}{3} \pi (\pi_3^3 - \pi_1^3)}{\rho \frac{4}{3} \pi (\pi_3^3 - \pi_1^3)}$$

ESERCIZIO 10

$$V(P_1) - V_{\infty} = V(P_1)$$



RICORDIAMO:

$$L_{A \rightarrow B} = \int_A^B \vec{F}_E \cdot d\vec{l} = q \int_A^B \vec{E} \cdot d\vec{l}$$

$$V_A - V_B = \int_A^B \vec{E} \cdot d\vec{l}$$

$$\Delta V = V_B - V_A = q(V_B - V_A) = q[-(V_A - V_B)]$$

a) $q_1 = 2 \text{ mC}$

b) $q_1 = 4 \text{ mC}$

$$V(P_1) = q_1 V(P_1)$$

Sapendo che $V(P_1) = \frac{1}{4\pi \epsilon_0} \frac{q_i}{|\vec{r}_1 - \vec{r}_i|}$

$$V_{q_1+q_2+q_3+q_4} = \frac{1}{4\pi \epsilon_0} \left[\frac{q_2}{|\vec{r}_1 - \vec{r}_2|} + \frac{q_3}{|\vec{r}_1 - \vec{r}_3|} + \frac{q_4}{|\vec{r}_1 - \vec{r}_4|} \right]$$

$$|\vec{r}_1 - \vec{r}_2| = 0,034 \text{ m} \quad |\vec{r}_1 - \vec{r}_4| = 0,017 \text{ m}$$

$$|\vec{r}_1 - \vec{r}_3| = \sqrt{0,034^2 + 0,017^2} = 0,038 \text{ m}$$

Dato:

$$V_{TOT} = \frac{1}{4\pi\epsilon_0} \left[\frac{25 \cdot 10^{-9} \text{ C}}{0,034 \text{ m}} + \frac{38 \cdot 10^{-9} \text{ C}}{0,038 \text{ m}} + \frac{32 \cdot 10^{-9} \text{ C}}{0,017 \text{ m}} \right] = \cancel{8,987 \cdot 10^4 \text{ V}}$$

$$= 8,987 \text{ N} \cdot \text{m}^2 \cdot \left[\frac{\text{C}}{\text{m}} \right] = 3,25 \cdot 10^4 \frac{\text{N} \cdot \text{m}}{\text{C}} = \text{J/C} = \text{V} = 3,25 \cdot 10^4 \text{ V}$$

Nel caso:

$$a) U = 2 \cdot 10^{-9} \text{ C} \cdot 3,25 \cdot 10^4 \text{ V} = 65 \mu\text{J}$$

$$b) U = 4 \cdot 10^{-9} \text{ C} \cdot 3,25 \cdot 10^4 \text{ V} = 130 \mu\text{J}$$

ESERCIZIO 11

$$q = 4 \mu\text{C} \quad \text{fra due punti a } \Delta V = 8 \text{ V}$$

$$\cancel{q} \Delta U = q \Delta V = 3,2 \cdot 10^{-4} \text{ J}$$

ESERCIZIO 12

$$V_{\text{elettronica}} = 4 \cdot 10^6 \text{ m/s} \quad E_{\text{in eV? (elettromvolt)}}$$

Elettromvolt = energ. pot. che un punto qualsiasi fra due punti a $1 \text{ V} = \Delta$

$$\cancel{1 \text{ eV} = 1,6 \cdot 10^{-19} \text{ J}}$$

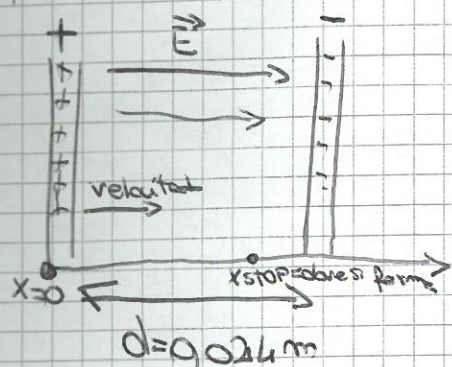
$$1 \text{ eV} = 1,6 \cdot 10^{-19} \text{ J}$$

$$E_{\text{cin}} = \frac{1}{2} m v^2 = 7,3 \cdot 10^{-18} \text{ J} = 45,5 \text{ eV}$$

ESERCIZIO 13

$$\Delta V = 100 \text{ V}$$

$$v_0 = 4 \cdot 10^6 \text{ m/s}$$



All'interno ci sarà un E uniforme

$$\vec{E} = E \hat{x} \text{ (direz. asse } x)$$

ci muoviamo
sux
↓
come sull'asse x
↓
 $dl = dx$

$$V_0 - V_d = \int_0^d \vec{E} \cdot d\vec{l} = \int_0^d E \hat{x} \cdot d\vec{l} \hat{x} = \int_0^d E \cdot dl = E \int_0^d dx$$

$\vec{E} = E \hat{x}$
 $d\vec{l} = d\vec{l} \hat{x}$

$\downarrow$
 il campo E uniforme
 cioè costante

$\downarrow$
 $= E \cdot d$

eh formula

$$\Delta V = \vec{E} \cdot d \rightarrow \vec{E} = \frac{100 \text{ V}}{0.024 \text{ m}} \cdot \hat{x}$$

in $x=0$

$$E_k = \frac{1}{2} m v^2$$

$$U = qV$$

~~x_{stop}~~

$$\Rightarrow \frac{1}{2} m v^2 + q V_0 = q V_{x_s}$$

$$\frac{1}{2} m v^2 = q (V_{x_s} - V_0)$$

Sappiamo che

$$V_{x_s} - V_0 = - \int_0^{x_s} \vec{E} \cdot d\vec{l} = - \int_0^{x_s} E dx = - E x_s$$

Abbiamo un elettrone quindi

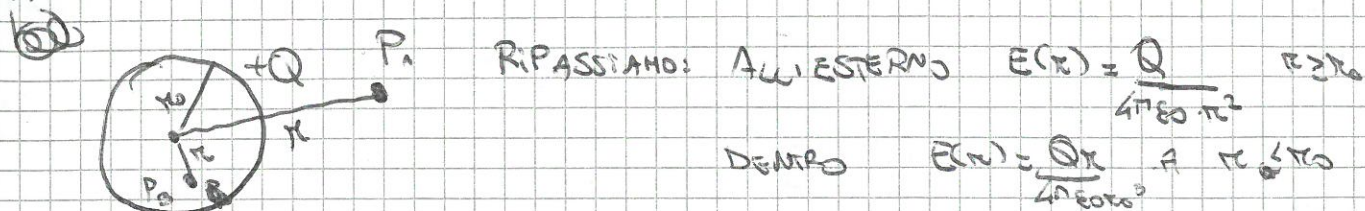
$$q = -e$$

$$\frac{1}{2} m v^2 = e \left[\frac{V_0 - V_d}{d} x_s \right]$$

$$x_{\text{stop}} = \frac{m v^2 \cdot d}{2e(V_0 - V_d)} = 0.014 \text{ m}$$

ESERCIZIO 14

~~$V(r) = ?$~~ all'esterno



$$a) V(r) - V(\infty) = \int_r^{\infty} \vec{E} \cdot d\vec{\ell} = \int_r^{\infty} \left(\frac{Q}{4\pi\epsilon_0 r'^2} \right) \cdot dr' \hat{r} = \frac{Q}{4\pi\epsilon_0} \int_r^{\infty} \frac{dr'}{r'^2}$$

$$= \frac{Q}{4\pi\epsilon_0} \left[-\frac{1}{r'} \right]_r^{\infty} = \frac{Q}{4\pi\epsilon_0 r}$$

$$b) V(r) - V_{\text{sup}} = \int_r^{r_0} \left(\frac{Qr'}{4\pi\epsilon_0 r_0^3} \right) \cdot (dr' \hat{r}) = \frac{Q}{4\pi\epsilon_0 r_0^3} \int_r^{r_0} r' dr' = \frac{Q}{4\pi\epsilon_0} \left[\frac{r'^2}{2} \right]_r^{r_0}$$

$$= \frac{Q}{8\pi\epsilon_0 r_0^3} [r_0^2 - r^2]$$

$$c) V(r_A) - V(r_B) = \int_{r_A}^{r_B} \vec{E} \cdot d\vec{\ell} = \int_{r_A}^{r_B} \frac{Q}{4\pi\epsilon_0 r^2} \hat{r} \cdot dr' \hat{r} = \frac{Q}{4\pi\epsilon_0} \int_{r_A}^{r_B} \frac{dr'}{r'^2}$$

$$= \frac{Q}{4\pi\epsilon_0} \left[-\frac{1}{r'} \right]_{r_A}^{r_B} = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{r_A} - \frac{1}{r_B} \right] = 60V$$

$$d) V(r_C) - V(r_0) = \int_{r_C}^{r_0} \frac{Qr'}{4\pi\epsilon_0 r_0^3} \cdot dr' \hat{r} = \frac{Q}{8\pi\epsilon_0 r_0^3} [r_0^2 - r_C^2] = 70V$$

ESERCIZIO 15

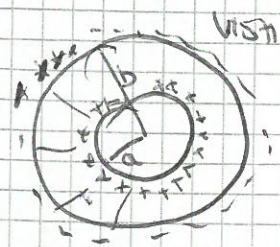
EZ620 N/C

Sappiamo che per un qualunque conduttore $\vec{E} = 0$ m

Da cui $\sigma_p = \epsilon_0 \cdot E_p = 8,85 \cdot 10^{-12} \frac{C^2}{N \cdot m^2} \cdot 620 \frac{N}{C} = 5,5 \frac{mC}{m^2}$

ESERCIZIO 16

(SIMILE A ESERCIZIO 8)



visi dall'alto

in b, quando sono conduttori

→ È UN CONDENSATORE CILINDRICO PRATICAMENTE

Dall'Es 8 sappiamo che $E(r) \neq 0$ solo in mezzo alle due superfici

a) $\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r} \hat{r}$ Per $a < r < b$ (da Es 8).

b) ΔV ? $Q = ?$

$$V_a - V_b = \int_a^b \left(\frac{\lambda}{2\pi\epsilon_0 r} \right) \cdot (d\pi r) = \frac{\lambda}{2\pi\epsilon_0} \int_a^b \frac{dr}{r} = \frac{\lambda}{2\pi\epsilon_0} [\ln r]_a^b =$$

$$= \frac{\lambda}{2\pi\epsilon_0} \ln \frac{b}{a} = V_a - V_b = \Delta V.$$

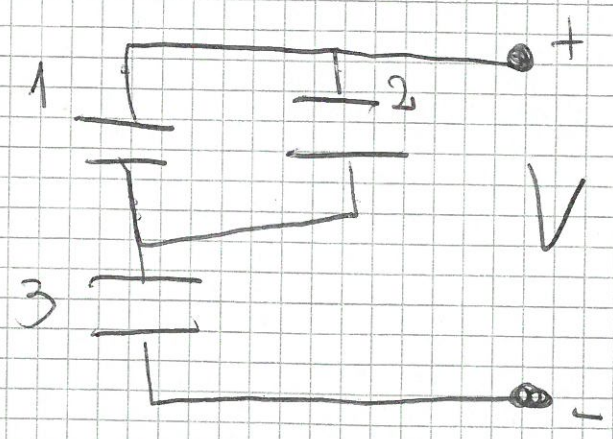
$C = \frac{Q}{\Delta V}$

Ora $Q = \lambda \cdot l$
lunghezza cilindro

da cui $C = \frac{\lambda l 2\pi\epsilon_0}{\ln \frac{b}{a}} = \frac{2\pi\epsilon_0}{\ln \frac{b}{a}}$

ESERCIZIO 17

$V = 200$ Volt $S = 100 \text{ cm}^2$ $l = 4 \text{ cm}$ $Q = ?$



$C = \epsilon_0 \cdot \frac{S}{d} = 8.85 \cdot 10^{-12} \cdot \frac{0.01}{0.04} = 22 \text{ pF}$
 È uguale per tutti e tre poiché geometrie uguali

1 e 2 sono in parallelo. → Stessa ΔV

→ $C_{12} = C_1 + C_2 = 2C$

C_{12} e C_3 sono in serie. → Stessa Q

$\frac{1}{C_{tot}} = \frac{1}{C_{12}} + \frac{1}{C_3} \rightarrow C_{tot} = \frac{2}{3} C$

Ora $Q_{tot} = C_{tot} \cdot \Delta V = \frac{2}{3} C \Delta V = 3 \text{ mC}$

↓
 Q_{tot}

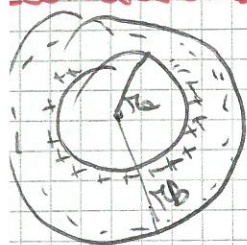
Tale carica $Q_{TOT} = Q_3 = Q_{12}$

$Q_3 = Q_{TOT}$

~~5. Qual è Q_{12}~~

Inoltre $Q_1 = Q_2 = \frac{Q_{12}}{2} = 15 \text{ mC}$.

ESERCIZIO 18



SFERA A PRIMA ARMATURA
SFERA B SECONDA ARMATURA

C?

$C = \frac{Q}{\Delta V}$ $\Delta V = V(a) - V(b) = \int_a^b \vec{E} \cdot d\vec{r}$

Il campo elettrico fra due superfici lo abbiamo già trovato in ES9

La ~~ES9~~ fra le sfere $E(r) = \frac{Q}{4\pi\epsilon_0 r^2}$

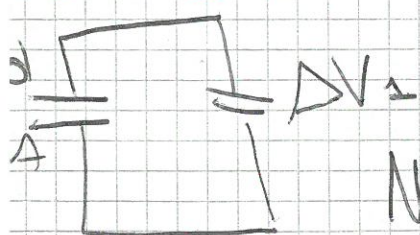
$$\Delta V = V(r_a) - V(r_b) = \int_{r_a}^{r_b} \vec{E} \cdot d\vec{r} = \int_{r_a}^{r_b} \frac{Q}{4\pi\epsilon_0 r^2} \cdot \hat{r} \cdot d\vec{r} = \frac{Q}{4\pi\epsilon_0} \int_{r_a}^{r_b} \frac{dr}{r^2} =$$

$$= \frac{Q}{4\pi\epsilon_0} \left[-\frac{1}{r} \right]_{r_a}^{r_b} = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{r_a} - \frac{1}{r_b} \right] = \frac{Q}{4\pi\epsilon_0} \frac{r_b - r_a}{r_a r_b} = \Delta V$$

$C = \frac{4\pi\epsilon_0}{\frac{r_b - r_a}{r_a r_b}} = \frac{4\pi\epsilon_0 r_a r_b}{r_b - r_a}$

QUESTE PRIMA DI ESSERE SCOLLEGATO

ESERCIZIO 19



$C_1 = \epsilon_0 \cdot \frac{A}{d}$ $Q_1 = C_1 \cdot \Delta V_1 = \epsilon_0 \cdot \frac{A}{d} \Delta V_1$

Noi sappiamo che $\frac{Q}{A} = \epsilon_0 E \rightarrow C = \frac{Q}{\Delta V} = \epsilon_0 \frac{A}{d}$

densità
sup cond $= k$

$E = \frac{Q}{\epsilon_0 A} = \frac{k}{\epsilon_0} = \text{costante} \rightarrow \text{anche dopo averle scollegate N.B.!!!}$

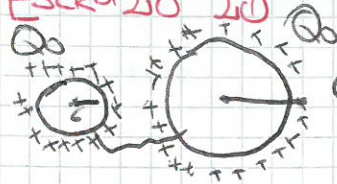
$\Delta V_2 = E \cdot 2d = 2\Delta V_1$ $C_2 = \epsilon_0 \cdot \frac{A}{\frac{2d}{2}} = \frac{1}{2} C_1$

$U_1 = \frac{1}{2} Q \Delta V_1 = \frac{1}{2} C_1 \Delta V_1^2 = \frac{1}{2} \epsilon_0 \frac{A}{d} \Delta V_1^2$ (PRIMA DISCOLL.)

$U_2 = \frac{1}{2} C_2 \Delta V_2^2 = \frac{1}{2} \epsilon_0 \frac{A}{\frac{2d}{2}} 4 \Delta V_1^2 = 2 U_1$ (DOPO SCOLL.)

Ora $L = U_2 - U_1 = 2U_1 - U_1 = U_1 = L$

ESERCIZIO 20



$$r_1 = 6 \text{ cm} \quad r_2 = 12 \text{ cm} \quad Q_0 = Q_1 = Q_2 = 3 \cdot 10^{-5} \text{ C}$$

$$\pi_2 = 2\pi_1$$

Collegare le sfere con un ~~cavo~~ conduttore significa fare un unico conduttore. Quindi la carica si distribuirà uniformemente sul conduttore totale. $\propto$ Sono due comoli di π .

$$① Q_{TOT} = 2Q_0 = Q_1 + Q_2$$

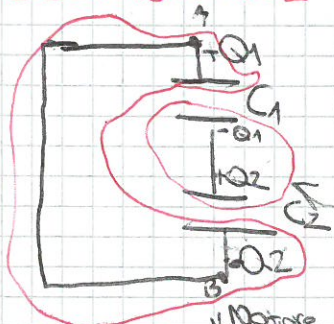
$$② V_1^{fin} = V_2^{fin} \Rightarrow \frac{Q_1^{fin}}{4\pi\epsilon_0 r_1} = \frac{Q_2^{fin}}{4\pi\epsilon_0 r_2} \Rightarrow \frac{Q_1^{fin}}{r_1} = \frac{Q_2^{fin}}{r_2} \Rightarrow \frac{Q_1^{fin}}{Q_2^{fin}} = \frac{r_1}{r_2} = \frac{1}{2} \Rightarrow Q_1^{fin} = \frac{1}{2} Q_2^{fin}$$

$$\text{Sappiamo che } V_{\text{sfera}} = \frac{Q}{4\pi\epsilon_0 R}$$

$$Q_{TOT} = Q_1 + Q_2 = Q_1 + 2Q_1 = 3Q_1 \rightarrow 2Q_0 = 3Q_1 \rightarrow Q_1 = \frac{2}{3} Q_0 = 2 \cdot 10^{-5} \text{ C}$$

$$Q_2 = 4 \cdot 10^{-5} \text{ C}$$

ESERCIZIO 21



$$C_1 = 2 \mu\text{F} \rightarrow Q_1 = C_1 \Delta V_0 = 48 \mu\text{C}$$

$$C_2 = 3C_1 \rightarrow Q_2 = C_2 \Delta V = 144 \mu\text{C}$$

$$\Delta V_0 = 24 \text{ V}$$

$\Rightarrow$ Sono le cariche dopo averli caricati collegandoli ad una batteria da 240 V

↓
dopo che si scollega la batteria e vengono messi insieme

↓
Nota che Pos di 1 è colto - di 2 è inverso.

SI FORMANO DUE CONDUTTORI.

La Q_{TOT} si conserva
e ogni punto è equipotenziale e centr.

$$1) Q = -Q_1 + Q_2 = 96 \mu\text{C}$$

$$\text{ESERVO } Q = -Q_2 + Q_1 = -96 \mu\text{C}$$

$$2) V_A - V_B = 0$$

$$\Delta V_1 = \frac{Q_1}{C_1}$$

$$\Delta V_2 = \frac{Q_2}{C_2}$$

$$\Rightarrow \frac{Q_1^{fin}}{C_1} = \frac{Q_2^{fin}}{C_2} \Rightarrow \frac{Q_1^{fin}}{2} = \frac{Q_2^{fin}}{3}$$

$$\text{Se } V_A - V_B = 0 \Rightarrow \Delta V_1 = \Delta V_2$$

Per la conservazione delle cariche

$$\text{tes } Q_{TOT} = -Q_1 + Q_2 = 96 \mu\text{C} = Q_{1, \text{centr}}^{fin} + Q_{2, \text{centr}}^{fin}$$

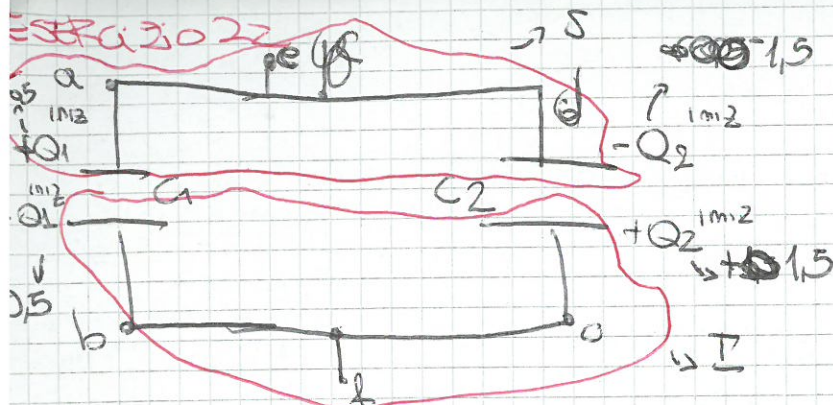
$$Q_2^{fin} = 3Q_1^{fin}$$

$$Q_{1, \text{centr}}^{fin} + Q_{2, \text{centr}}^{fin} = 96 \mu\text{C}$$

$$\rightarrow 4Q_{1, \text{centr}}^{fin} = 96 \mu\text{C}$$

$$Q_{1, \text{centr}}^{fin} = 24 \mu\text{C}$$

$$Q_{2, \text{centr}}^{fin} = 72 \mu\text{C}$$



$$\Delta V = \frac{Q}{C} \Rightarrow Q = \Delta V \cdot C =$$

$$1) Q_{TOT}^S = Q_1^{lim} - Q_2^{lim} = -2 \cdot 10^{-4} C = Q_{1,S}^{lim} + Q_{2,S}^{lim}$$

$$Q_{TOT}^I = -Q_1^{lim} + Q_2^{lim} = 2 \cdot 10^{-4} C = Q_{1,I}^{lim} + Q_{2,I}^{lim}$$

$$2) V_a = V_d \Rightarrow \Delta V_1^{lim} = \Delta V_2^{lim} \Rightarrow \frac{Q_1^{lim}}{C_1} = \frac{Q_2^{lim}}{C_2} \Rightarrow Q_2^{lim} = 3 Q_1^{lim}$$

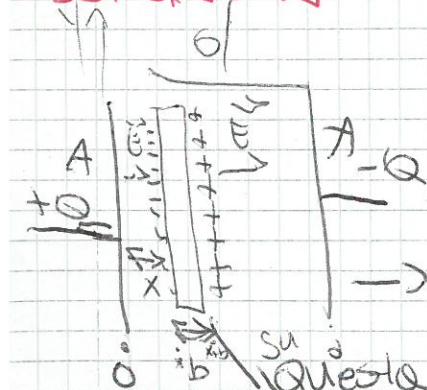
$$Q_2^{lim} = 3 Q_1^{lim}$$

$$\begin{cases} Q_{1,S}^{lim} + Q_{2,S}^{lim} = -2 \cdot 10^{-4} C \end{cases}$$

$$4 Q_{1,S}^{lim} = -2 \cdot 10^{-4} C \rightarrow Q_{1,S}^{lim} = -0.5 \cdot 10^{-4} C$$

$$Q_{2,S}^{lim} = -1.5 \cdot 10^{-4} C$$

ESERCIZIO 2.3



$$a) C = \epsilon_0 \cdot \frac{A}{d}$$

→ PRATICAMENTE SI FORMANO DUE CONDENSATORI IN SERIE

su questa lastra si accumulano cariche negative e si misre cariche positive

$$b) E \text{ è uguale in ogni punto } \vec{E} = \frac{\sigma}{\epsilon_0} \hat{x} \quad \sigma = \frac{Q}{A}$$

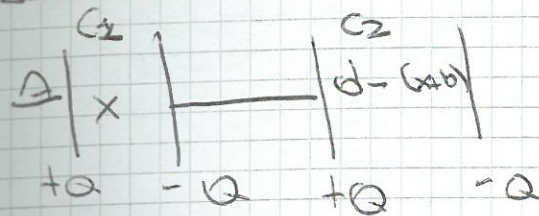
Se inseriamo la barretta

$$\Delta V = \int_0^x \vec{E} \cdot d\vec{x} + \int_{x+b}^d \vec{E} \cdot d\vec{x} = \frac{\sigma}{\epsilon_0} [x+d - x+b] = \frac{\sigma}{\epsilon_0} (d+b)$$

$$\text{Quindi: } C = \frac{Q}{\Delta V} = \frac{\sigma A}{\frac{\sigma}{\epsilon_0} (d+b)} = \frac{\epsilon_0 \cdot A}{d+b}$$

PER FARE UN ALTRO METODO

SELI CONSI DERIAMO COME DUE CONDENSATORI IN SERIE



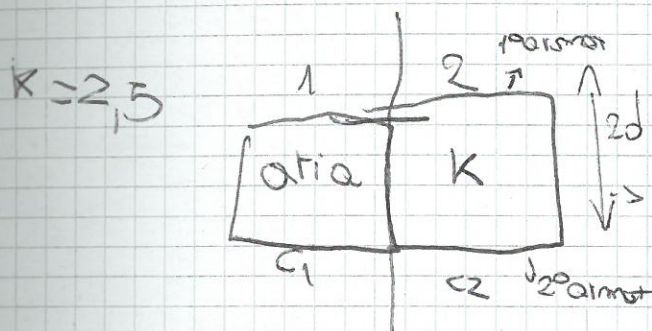
$$C_1 = \epsilon_0 \frac{A}{x}$$

$$C_2 = \epsilon_0 \frac{A}{d-(x+b)}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{x}{\epsilon_0 A} + \frac{d-(x+b)}{\epsilon_0 A} = \frac{d-b}{\epsilon_0 A} \rightarrow \boxed{C = \frac{\epsilon_0 A}{d-b}}$$

UN METODO PIU' VELOCE, STESSO RISULTATO

ESERCIZIO 24



$$C_1 = \epsilon_0 \frac{A}{2d}$$

$$C_2 = K C_1 = K \epsilon_0 \frac{A}{2d}$$

$$U_{TOT} = \frac{1}{2} C \Delta V^2$$

~~Capacitor~~

È come se fossero due cond. in parallelo

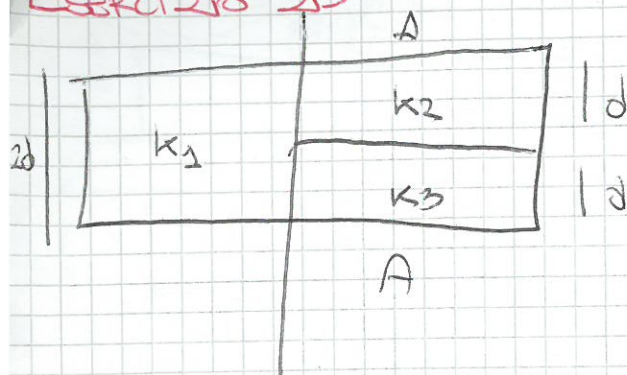
$$C_{TOT} = C_1 + C_2 = \epsilon_0 \frac{A}{2d} (1+K)$$

$$U_{TOT} = \frac{1}{2} C_{TOT} \Delta V^2 = \frac{1}{2} \epsilon_0 \frac{A}{2d} (1+K) \Delta V^2$$

$$U_2 = \frac{1}{2} C_2 \Delta V^2 = \frac{1}{2} K \epsilon_0 \frac{A}{2d} \Delta V^2$$

$$\frac{U_2}{U_{TOT}} = \frac{K}{1+K} = 0.7 \approx 70\%$$

Esercizio 2.5



Lo possiamo vedere come tre condensatori

$$C_1 = k_1 C_1 = k_1 \epsilon_0 \frac{A}{2d}$$

$$C_2 = k_2 \cdot \epsilon_0 \frac{A}{2d}$$

$$C_3 = k_3 \epsilon_0 \frac{A}{2d}$$

2 e 3 sono in serie

$$\frac{1}{C_{23}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{2d}{\epsilon_0 A} \left(\frac{1}{k_2} + \frac{1}{k_3} \right) = \frac{2d}{\epsilon_0 A} \frac{k_2 + k_3}{k_2 \cdot k_3}$$

$$C_{TOT} = C_1 + C_{23} = \epsilon_0 \frac{A}{2d} \left(\frac{k_1}{2} + \frac{k_2 k_3}{k_2 + k_3} \right)$$

perall.

si considera comunque

2d e non d

perché le piastre

sono comunque a 2d

e k1 e k2 sono comunque

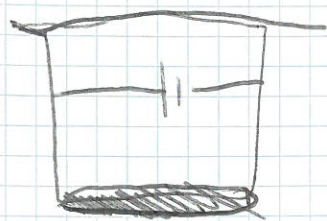
rispetto a 2d

↑
N.B.!!!

ESERCIZIO 1 (Magnetismo)

$$\lambda = 0,04 \text{ kg/m} \cdot \vec{e}$$

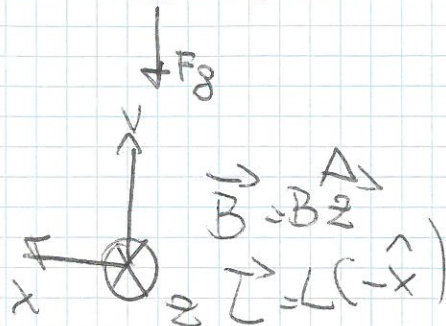
$$B = 3,6 \text{ T}$$



$$\begin{cases} \vec{F}_g = mg(-\hat{y}) \\ \vec{F}_B = I\vec{L} \times \vec{B} = ILB(\hat{y}) \end{cases}$$

$$\int \lambda g(-\hat{y}) \cdot d\vec{L} = ILB$$

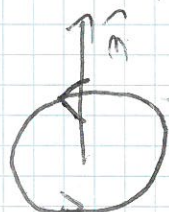
$$I = \frac{\lambda g}{B} = 0,11$$



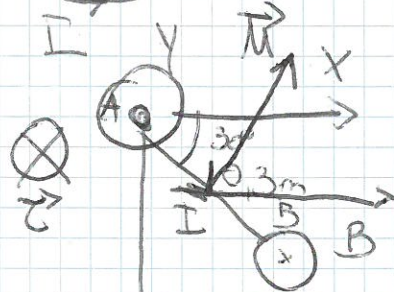
ESERCIZIO 2

$$N = 100 \text{ spire} \quad a = 0,4 \text{ m} \quad b = 0,3 \text{ m} \quad \tau = ?$$

$$B = 0,8 \text{ T} \quad I = 1,2 \text{ A}$$



$$\vec{\mu} = I\vec{A} = IA\hat{n}$$



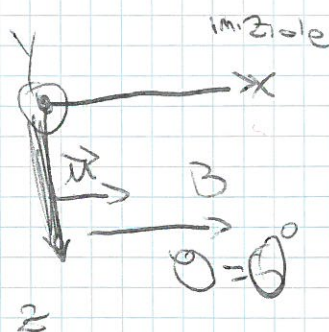
$$AB = 0,3 \text{ m} = l$$

$$\vec{B} = B\hat{x}$$

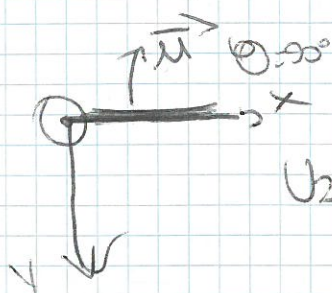
$$\vec{\mu} = N \cdot I A \hat{n} = I \cdot ab \cdot N$$

$$\vec{\tau} = \vec{\mu} \times \vec{B} = N I a b \sin(\theta) \cdot \hat{y}$$

$$\tau = 10 \text{ N} \cdot \text{m}$$



$$U_1 = -\vec{\mu} \cdot \vec{B} = -N I a b B = -11,55$$



$$U_2 = -\vec{\mu} \cdot \vec{B} = 0$$

$$\Delta U = 11,55$$

ESERCIZIO 4

$$E = 100 \text{ eV} \quad B = 35 \mu\text{T}$$

$$m_{\text{elet}} = 9,1 \cdot 10^{-31} \text{ kg}$$

$$100 \cdot 10^{-19} \text{ J}$$

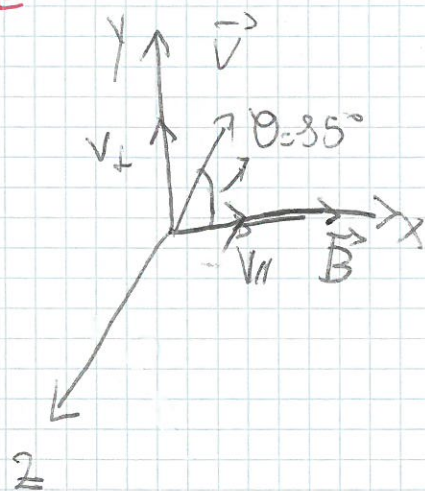
$$\lambda = \frac{h}{p} = \frac{h}{m v} = \frac{6,6 \cdot 10^{-34} \text{ J}\cdot\text{s}}{9,1 \cdot 10^{-31} \text{ kg} \cdot 1,6 \cdot 10^{-9} \text{ m/s}} = 0,978 \cdot 10^{-6} \text{ m} = 978 \text{ nm}$$

per la legge di Newton $m a_c = \frac{m v^2}{r}$

$$\vec{F}_B = -e \vec{v} \times \vec{B} \Rightarrow |\vec{F}_B| = -e v_{\perp} B = m \frac{v_{\perp}^2}{r} \Rightarrow r = \frac{m v_{\perp}}{e B} = 0,965$$

$$v_{\perp} = \sqrt{\frac{2 E_K}{m}} \approx 6 \cdot 10^6 \text{ m/s}$$

ESERCIZIO 5



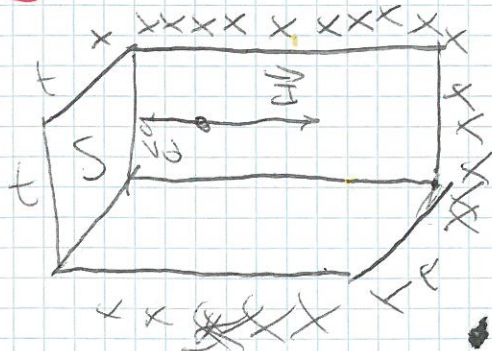
$$\vec{v} = \vec{v}_{\perp} + \vec{v}_{\parallel}$$

$$\vec{F}_B = e (\vec{v}_{\perp} \times \vec{B}) \Rightarrow |\vec{F}_B| = e v_{\perp} B = e v \sin \theta B = \frac{m (v \sin \theta)^2}{r}$$

$$r = \frac{m v \sin \theta}{e B} \approx 2 \cdot 10^{-4} \text{ m}$$

$$p = v_{\parallel} \cdot T = v_{\parallel} \cdot \frac{1}{f} = v \cos \theta \frac{2\pi m}{e B} \approx 10^{-4} \text{ m}$$

ESERCIZIO 3



$$\vec{v}_d = v_d (-\hat{y})$$

$$n = 848 \cdot 10^{23} \frac{\text{e}}{\text{m}^3}$$

$$\vec{F}_B = -e n_d \vec{v}_d \times \vec{B} = -e n_d v_d B (-\hat{y}) \times (-\hat{z}) = e n_d v_d B \hat{x}$$

$$\vec{F}_E = -e E_H \hat{x} = e E_H (-\hat{x})$$

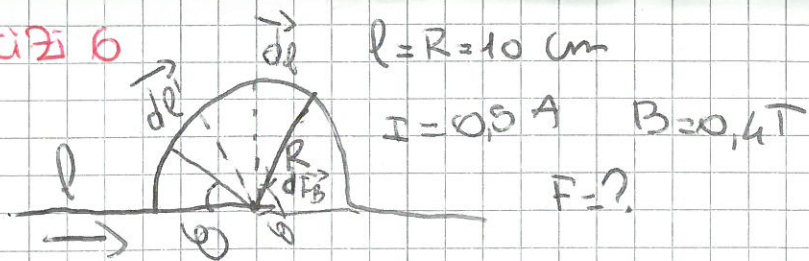
distanza lorentz

$$\Delta V_H = \frac{I}{n e} B$$

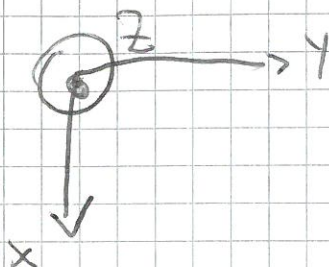
Equilibrio $F_B = F_E \Rightarrow$ MISURA TENSIONE DI HALL ΔV_H

$$e n_d v_d = e E_H \Rightarrow E_H = v_d B \Rightarrow E_H = \frac{I}{n e w} B \Rightarrow \text{campo di Hall all'equilibrio}$$

ESERCIZIO 6



FISSIAMO GLI ASSI



SUTTRATTI DI FILI RETTILINEI

$$\vec{F}_B = I \vec{l} \times \vec{B} = I (R \hat{y}) \times (B \hat{z}) = I R B \hat{x}$$

SULLA SEMICIRC.

$$d\vec{F}_B = I d\vec{l} \times \vec{B} \cdot \hat{n} = I dl \cdot B (\cos\theta (-\hat{y}) + \sin\theta \hat{x})$$

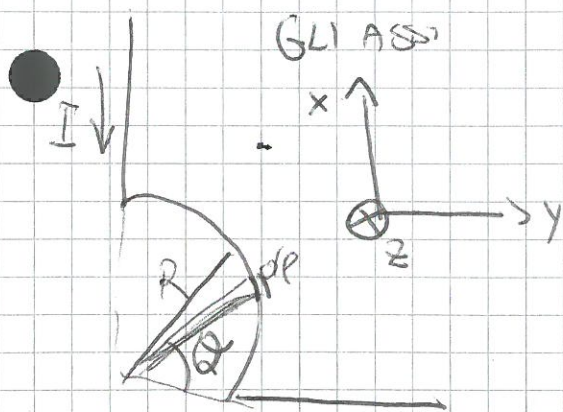
$$d\vec{F}_B = I dl B (\cos\theta \hat{y} + \sin\theta \hat{x})$$

$$\Rightarrow dF_{Bx} = I B R d\theta \sin\theta$$

$$\vec{F}_B^{\text{semicirc.}} = \hat{x} I R B \int_0^\pi \sin\theta d\theta = \hat{x} I R B [-\cos\theta]_0^\pi = 2 I R B \hat{x}$$

$$\vec{F}_B^{\text{TOT}} = 2 I R B \hat{x} + 2 I R B \hat{x} = 4 I R B \hat{x} = (0.08 \text{ N}) \hat{x}$$

ESERCIZIO 7



$$d\vec{B} = \frac{\mu_0 \cdot I}{4\pi} \cdot \frac{d\vec{l} \times \vec{r}}{r^2}$$

$$\vec{B}_{\text{filo}} = \frac{\mu_0}{4\pi} I \cdot dl \cdot \frac{\sin\theta}{r} = 0 \quad \text{perché } dl \perp r \text{ quindi } \sin\theta = 0$$

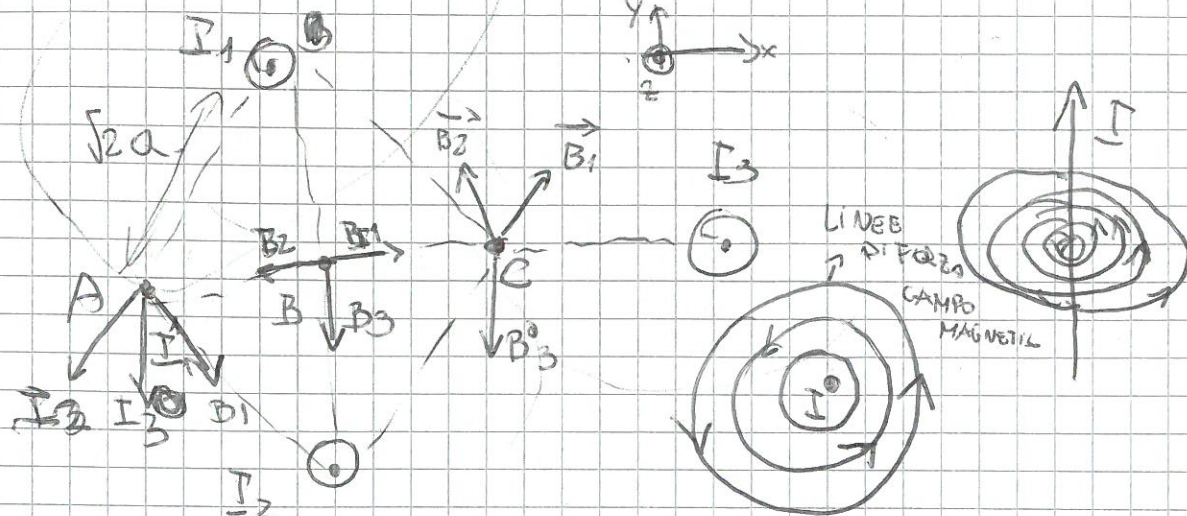
MA PER IL FILO SOTTO CHE SOPRA

$$d\vec{B}_{\text{arco}} = \frac{\mu_0}{4\pi} I \cdot \frac{dl \times \vec{r}}{R^2} = \frac{\mu_0 \cdot I}{4\pi} \frac{dl}{R^2} \hat{z} = \frac{\mu_0 \cdot I}{4\pi} R \cdot d\theta \frac{\hat{z}}{R^2} = \frac{2}{4\pi} \frac{\mu_0 \cdot I}{R} d\theta \hat{z}$$

$$\vec{B}_{\text{arco}} = \frac{\mu_0 I}{4\pi R} \int_0^\pi d\theta = \frac{\mu_0 \cdot I}{4\pi R} \cdot \frac{\pi}{2} \approx 0.20 \text{ G}$$

ESERCIZIO 8

SISTEMA



$$\vec{B}_1(A) = \frac{\mu_0 \cdot I}{2\pi\sqrt{2}a} [\cos\frac{\pi}{4} \hat{x} + \sin\frac{\pi}{4} (-\hat{y})]$$

$$\vec{B}_2(A) = \frac{\mu_0 \cdot I}{2\pi\sqrt{2}a} [\cos\frac{\pi}{4} (-\hat{x}) + \sin\frac{\pi}{4} (-\hat{y})]$$

$$\vec{B}_3(A) = \frac{\mu_0 \cdot I}{2\pi a} (-\hat{y})$$

$$\vec{B}_{TOT} = \vec{B}_1(A) + \vec{B}_2(A) + \vec{B}_3(A) = -\hat{y} \left(\frac{\mu_0 \cdot I}{2\pi a} \right) \left(\frac{1}{2} + \frac{1}{2} + 1 \right) = -\hat{y} \left(\frac{2\mu_0 \cdot I}{\pi a} \right)$$



SUB

$$\vec{B}_1(B) = \frac{\mu_0 \cdot I}{2\pi a} (\hat{x})$$

$$\vec{B}_2(B) = \frac{\mu_0 \cdot I}{2\pi a} (-\hat{x})$$

$$\vec{B}_3(B) = \frac{\mu_0 \cdot I}{4\pi a} (-\hat{y})$$

$$\vec{B}_{TOT} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$$



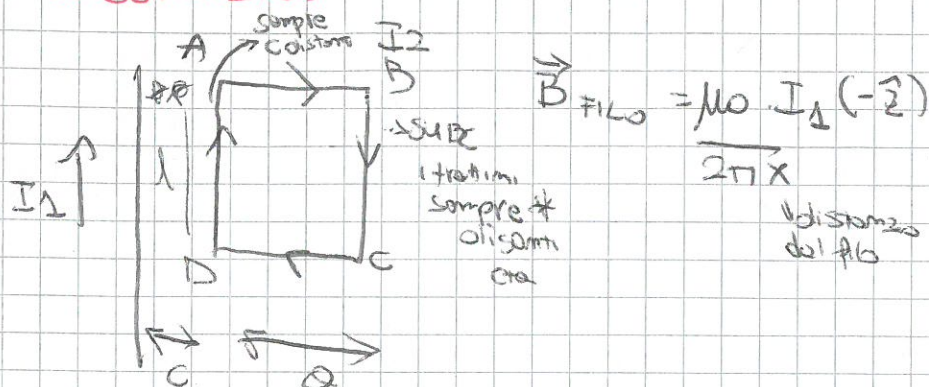
SUC

$$\vec{B}_1(C) = \frac{\mu_0 \cdot I}{2\pi\sqrt{2}a} [\cos\frac{\pi}{4} \hat{x} + \sin\frac{\pi}{4} \hat{y}] \quad \vec{B}_2(C) = \frac{\mu_0 \cdot I}{2\pi\sqrt{2}a} [\cos\frac{\pi}{4} (-\hat{x}) + \sin\frac{\pi}{4} \hat{y}]$$

$$\vec{B}_3(C) = \frac{\mu_0 \cdot I}{2\pi a} (-\hat{y})$$

$$\vec{B}(0) = \frac{\mu_0 I}{2\pi a} \left(\frac{1}{2} + \frac{1}{2} - 1 \right) = 0$$

ESERCIZIO 9



$$\vec{B}_{FIL0} = \frac{\mu_0 I_1}{2\pi x} (-\hat{z})$$

$\downarrow$
distanza dal filo

SU AB $d\vec{F}_B = I_2 d\vec{l} \times \vec{B}_{FIL0} = \hat{y} I_2 dx B_{FIL0}(x)$

$\downarrow$ $\downarrow$
 $dx \cdot \hat{x}$ $B_{FIL0}(-\hat{z})$

$$\vec{F}_{AB} = \hat{y} I_2 \int_c^{a+c} \frac{\mu_0 I_1}{2\pi x} dx = \hat{y} \frac{\mu_0 I_1 I_2}{2\pi} \int_c^{a+c} \frac{dx}{x} =$$

$$\hat{y} \frac{\mu_0 I_1 I_2}{2\pi} \ln\left(\frac{a+c}{c}\right)$$

SU DC $d\vec{F}_B = I_2 d\vec{l} \times \vec{B}_{FIL0} = (-\hat{y}) I_2 dx B_{FIL0}(x)$

$\downarrow$ $\downarrow$
 $dx \cdot \hat{x}$ $B(-\hat{z})$

$$F_{CD} = (-\hat{y}) I_2 \int_c^{a+c} \frac{\mu_0 I_1}{2\pi x} dx = (-\hat{y}) \frac{\mu_0 I_1 I_2}{2\pi} \int_c^{a+c} \frac{dx}{x}$$

$$= (-\hat{y}) \frac{\mu_0 I_1 I_2}{2\pi} \ln\left(\frac{a+c}{c}\right)$$

SU BC $\vec{F}_{BC} = I_2 \vec{l} \times \vec{B}_{FIL0}(a+c) = \hat{x} \frac{\mu_0 I_1 I_2 \cdot l}{2\pi(a+c)}$

$\downarrow$ $\downarrow$
 $l \cdot (-\hat{y})$ $B(-\hat{z})$

$\frac{\mu_0 I_1}{2\pi(a+c)}$

A

SU AD $\vec{F}_{AD} = I_2 \vec{l} \times \vec{B}_{FIL0}(c) = (-\hat{x}) \frac{\mu_0 I_1 I_2 \cdot l}{2\pi c}$

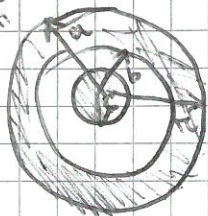
$\downarrow$ $\downarrow$
 $l \cdot \hat{y}$ $B(-\hat{z})$

$\frac{\mu_0 I_1}{2\pi c}$

$$\vec{F}_{TOT} = \frac{\mu_0 \cdot I_1 \cdot I_2 \cdot l}{2\pi} \left(\frac{1}{a} - \frac{1}{b} \right) \hat{x} = -2.7 \cdot 10^{-5} \text{ N} \cdot \hat{x}$$

Esercizio 10

LA CORRENTE
È SOLO IN QUANTITÀ
TRANSVERSALE



$B(r)$?

- 1) $r \leq c$ 2) $c \leq r \leq b$ 3) $b < r \leq a$ 4) $r > a$

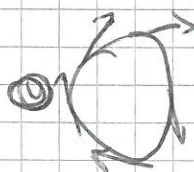
Allora:

$$\vec{B}(r) = B(r) \hat{e}$$

$\downarrow$
dir. tangente



→ verso positivo



→ corrente negativa

$$\oint \vec{B} \cdot d\vec{e} = \mu_0 \cdot I_{conc}$$

$\downarrow$

$$\oint_{\Delta \pi c(m)=2\pi r} B(r) \hat{e} \cdot dl \hat{e} = \oint_{2\pi r} B(r) dl = B(r) \oint_{2\pi r} dl = \mu_0 I_{conc}$$

Dalci:

$$B(r) = \frac{\mu_0 I_{conc}}{2\pi r}$$

1) $r \leq c$ $I_{conc} = \frac{I}{\pi \cdot c^2} \cdot \pi r^2 = \frac{I \cdot r^2}{c^2}$

$\downarrow$
 J = densità di corrente

$$\Downarrow$$

$$B(r) = \frac{\mu_0 \cdot I \cdot r^2}{2\pi r c^2} = \frac{\mu_0 \cdot I \cdot r}{2\pi c^2} \Rightarrow \vec{B}(r) = \frac{\mu_0 I r}{2\pi c^2} \hat{e}$$

2) $c < r \leq b$

La I_{conc} è tutta quella in c , quindi I

$$B(r) = \frac{\mu_0 \cdot I}{2\pi r}$$

3) $b < \pi < a$. $I_{\text{conc}} = \frac{I}{\pi(a^2 - b^2)}$ $\cdot \pi(\pi^2 - b^2)$

$\downarrow$
 $I = \text{dens. corrente}$

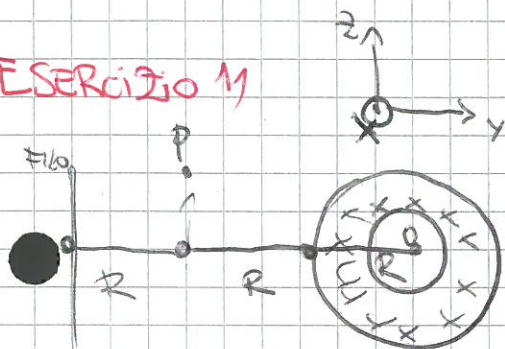
$\vec{B}(x) = \frac{\mu_0 I}{2\pi x} \left(\frac{a^2 - \pi^2}{(a^2 - b^2)} \right) \hat{z}$

4) $\pi > a \rightarrow I_{\text{conc}} = 0 \Rightarrow \boxed{B(x) = 0}$

PER LO REGOLA MANO DESTRA

QUINDI I DEVE ESSERE VERSO IL BASSO

ESERCIZIO 11



a) I_{Filo} all'infinito $B(P) = 0$?

$\vec{B}_{\text{filo}}(P) = B_{\text{filo}}(-\hat{z})$

QUINDI $\vec{B}_{\text{filo}} = \frac{\mu_0 \cdot I_{\text{filo}}}{2\pi R} = \frac{\mu_0 I_{\text{tot}}}{4\pi R} = B_T$

$\boxed{I_F = \frac{I_T}{2}}$

$\frac{1}{2} + \frac{1}{6} = \frac{1}{3}$

b) $\vec{B}_{\text{tot}}(P) = -B(0)$

$\vec{B}_T(0) = 0$
 $\vec{B}_F(0) = \frac{\mu_0 I_F}{6\pi R} (-\hat{z})$

$\vec{B}_T(P) = \frac{\mu_0 \cdot I_T}{4\pi R} \hat{z}$

$B_F(P) = \frac{\mu_0 I_F}{2\pi R (-\hat{z})}$

$\boxed{I_F = \frac{3}{4} I_T}$

$\frac{\mu_0 \cdot I_T}{4\pi R} - \frac{\mu_0 \cdot I_F}{2\pi R} = \frac{\mu_0 \cdot I_F}{6\pi R}$

$\frac{\mu_0 I_T}{4\pi R} = \frac{\mu_0 I_F}{3\pi R} \Rightarrow \frac{I_F}{3} = \frac{I_T}{4}$

ESERCIZIO 12

$$R = 50 \text{ cm} \quad r = 2 \text{ mm} \quad l = 10 \text{ m} \quad \rho = 1.7 \cdot 10^{-8} \Omega \cdot \text{m}$$

$$\mathcal{E} = 20 \text{ V}$$

$$\text{Resistenza filo} = \frac{\rho l}{\pi r^2} = 1.05 \cdot 10^{-2} \Omega \quad I = \frac{\mathcal{E}}{R_{\text{filo}}} \Rightarrow B_{\text{sol}} = \mu_0 I$$

a) $B_{\text{sol}} = ?$ se $\mathcal{E} = 20 \text{ V}$

$$m = \frac{N_{\text{spire}}}{L_{\text{sol}}} = \frac{l}{2\pi R} = \frac{1}{2\pi} \text{ spira/metro}$$

$$B = \mu_0 I = \frac{1}{2\pi} \mu_0 \frac{\mathcal{E}}{R_{\text{filo}}} = 0.46 \text{ T}$$

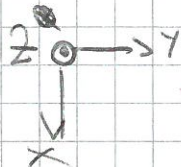
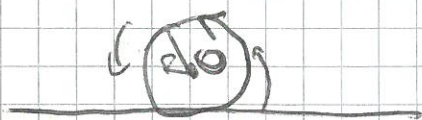
b) momento magnetico

$$\vec{\mu} = I S \cdot \hat{z} \quad \boxed{\tau = \vec{\mu} \times \vec{B} = 0} \quad \text{perché } \vec{\mu} \parallel \vec{B}$$

c) se $\vec{\mu} \perp \vec{B}$ $\tau = \vec{\mu} \times \vec{B} = (-1) m \mu_0 \cdot I \cdot I S \hat{z} \approx 3.7 \cdot 10^{-5} \text{ N}\cdot\text{m}$
 $\downarrow$
 $3.7 \cdot 10^{-5} \text{ N}\cdot\text{m}$

ESERCIZIO 13

$$I = 10 \text{ A} \quad R = 10 \text{ cm}$$



NEL PUNTO O

$$\vec{B}_{\text{filo}}(O) = \mu_0 \cdot I \cdot \hat{z} \quad \text{perché } \vec{B}_{\text{filo}}(O) = \frac{\mu_0 I}{2\pi R}$$

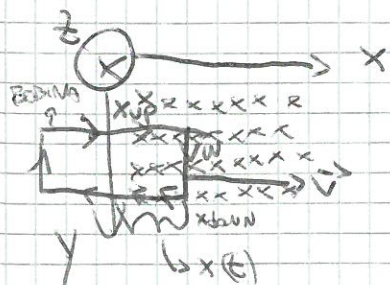
$$\vec{B}_{\text{spira}}(O) = \frac{\mu_0 I}{2R} \hat{z}$$

in un punto x $B(x) = \mu_0 \cdot \frac{I^2}{2\pi R^2} \times \frac{3}{2}$

ORA:

$$\vec{B}_{\text{tot}}(O) = \frac{\mu_0 I}{2R} \left(1 + \frac{1}{\pi}\right) \hat{z} = \boxed{8.3 \cdot 10^{-5} \text{ T}}$$

ESERCIZIO 1



$$\vec{B} = B \hat{z}$$

$$\vec{v} = v \hat{x}$$

$$\Phi_B = \int \vec{B} \cdot d\vec{S} = NB \cdot l \cdot x(t)$$

Spira

SAPPIAMO CHE:

$$\text{f.e.m. } \mathcal{E} = - \frac{d\Phi_B}{dt} = -NB \left(\frac{dx(t)}{dt} \right) = -NBv$$

Per la legge di Ohm $I = \frac{-NBv}{R}$

a) LE FORZE MAGNETICHE

SI AVVICINA QUINDI VERSO CORRENTE ANTICLOCKWISE

$$\vec{F}_B(x_{up}) = I \vec{x}(t) \times \vec{B} = I \cdot l \cdot B \hat{y}$$

-x z

UGUALI E OPPOSITE

$$\vec{F}_B(x_{down}) = I \vec{x}(t) \times \vec{B} = I \cdot l \cdot B (-\hat{y})$$

-x z

$$F_B(l) = I \vec{w} \times \vec{B} = I w B (-\hat{x})$$

-x z

b) $\Phi_B = \int \vec{B} \cdot d\vec{S} = NBwl$

$$\frac{d\Phi_B}{dt} = 0$$

$\mathcal{E} = 0$
 $I = 0$
 $\vec{F}_B = 0$

perché non c'è variazione tempo

c) $\Phi_B = \int \vec{B} \cdot d\vec{S} = NBw x(t)$

derivata di x(t) è velocità

S. ALLONTANA

QUI LA CORRENTE GIRA IN VERSO ORARIO

$$\mathcal{E} = - \frac{d\Phi_B}{dt} = -NBwv$$

$$I = \frac{|\mathcal{E}|}{R} = \frac{NBwv}{R}$$



$$\vec{F}_B(l) = I \vec{w} \times \vec{B} = I w B (-\hat{x})$$

w-y

ESERCIZIO 2

$$\vec{B} = B \hat{z}$$

$$I(t) = I_0 + \alpha t$$

$$\alpha = 250 \text{ A/s}$$

$$B(t) = \mu_0 I(t) = \mu_0 (I_0 + \alpha t)$$

$$a) \Phi_B(t) = \oint \vec{B} \cdot d\vec{s} = B(t) \pi r_s^2 = \pi r_s^2 \mu_0 (I_0 + \alpha t)$$



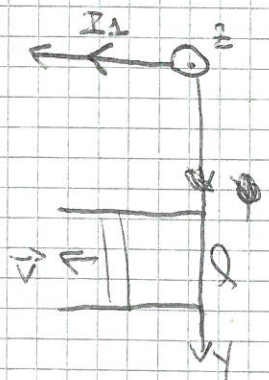
IL FLUSSO AUMENTA $\Rightarrow$ QUINDI CORRENTE IN ANELLA CIRCOLA IN VERSO OPPOSTO ALLE SPIRE DEL SOLENOIDE, PERCHÉ SI OPPONE

$$\mathcal{E}_i = - \frac{d\Phi_B}{dt} = -\pi r_s^2 \mu_0 \alpha \Rightarrow I_i = \frac{|\mathcal{E}_i|}{R_i} = \frac{\pi r_s^2 \mu_0 \alpha}{R} = 3,2 \text{ A}$$

al centro

$$b) \vec{B}(\vec{r}) = \frac{\mu_0 I}{2\pi R_{\text{anello}}} (-\hat{z})$$

ESERCIZIO 3



$$\vec{V} = V \hat{x}$$

$$\vec{B}_{filo} = B \hat{z} = \frac{\mu_0 I_2}{2\pi y} \hat{z}$$

$$\mathcal{E} = - \frac{d\Phi_B}{dt}$$

$$\Phi_B = \int \vec{B} \cdot d\vec{s} = \int_{\text{spira}} B(y) dx dy = \int_0^{x(t)} dx \int_a^{a+l} B(y) dy =$$

$$= \frac{\mu_0 I_2}{2\pi} x(t) \left[\ln y \right]_a^{a+l} = \frac{\mu_0 I_2}{2\pi} x(t) \ln \left(\frac{a+l}{a} \right)$$

$$a) \mathcal{E} = - \frac{d\Phi_B}{dt} = - \frac{\mu_0 I_2}{2\pi} \ln \left(\frac{a+l}{a} \right) V = -2,5 \cdot 10^{-4} \text{ V}$$

$$b) I = \frac{|\mathcal{E}|}{R} = 6 \cdot 10^{-4} \text{ A}$$

$$c) P = R \cdot I^2 = 1,5 \cdot 10^{-7} \text{ W}$$

$$d) \oint \vec{F}_B = I \cdot \underbrace{d\vec{y}}_{dy(-\hat{y})} \times \underbrace{\vec{B}(y)}_{B(y)\hat{z}} = (-\hat{x}) I B(y) dy$$

$$\vec{F}_B = (-\hat{x}) \frac{\mu_0 I_1 I}{2\pi} \int_a^{a+l} \frac{dy}{y} = (-\hat{x}) \frac{\mu_0 \cdot I_1 \cdot I}{2\pi} \ln\left(\frac{a+l}{a}\right)$$

$$\vec{F}_B^{\text{ESTERNO}} = \hat{x} \frac{\mu_0 \cdot I_1 \cdot I}{2\pi} \ln\left(\frac{a+l}{a}\right) = (3,2 \cdot 10^{-8} \text{ N}) (\hat{x})$$

$$e) P = \vec{F} \cdot \vec{v} = 1,5 \cdot 10^{-2} \text{ W}$$

ESERCIZIO 5

1) A_1, m_1

2) $A_2 > A_1, m_2$

coeff. mutua induttiva

$$\oint_{21} (B) = B_1 m_2 l A_1 = m_1 \mu_0 I_1 m_2 l A_1 = M_{21} I_1$$

$$M_{21} = \mu_0 m_1 m_2 l A_1$$

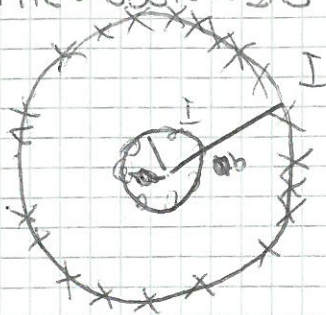
$$\oint_{12} (B) = B_2 m_1 l A_1 = m_2 \mu_0 I_2 m_1 l A_1 = M_{12} I_2$$

coeff. mutua induttiva

$$M_{12} = \mu_0 m_1 m_2 l A_1$$

ESERCIZIO 6. a)

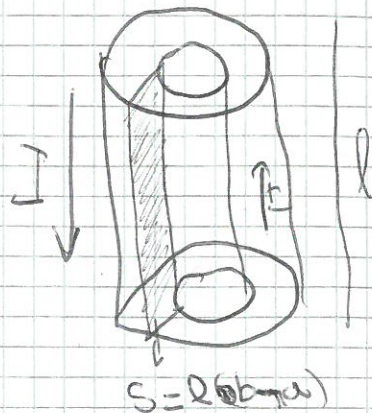
(SIMILE ESERCIZIO 10 ARGOMENTO PRIMA)



$$B(r) = \frac{\mu_0 I}{2\pi r} \quad a \leq r \leq b$$

$\rightarrow$ t. antiorario

$$\oint B = LI$$



Quindi $\oint B = \int \vec{B} \cdot d\vec{S} = \int B(r) dS = \int_0^l dz \int_a^b B(r) dr =$

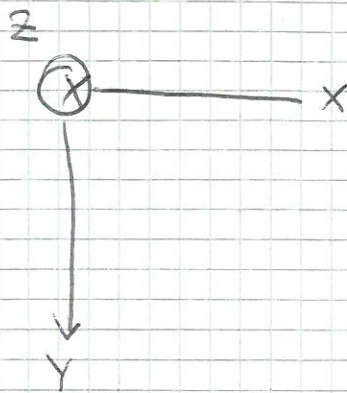
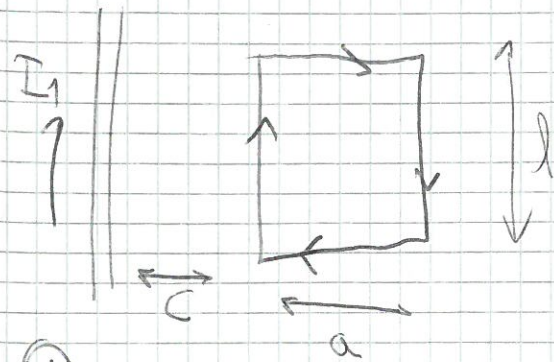
$\downarrow$ $\downarrow$
 $B(r) \hat{e}_\phi$ $dS \hat{m}$
 $\parallel$
 $d\pi \cdot dz$
 $\downarrow$ $\downarrow$
 dist. dall'asse centrale dovrebbe essere il

$$= \frac{\mu_0 I}{2\pi} l \int_a^b \frac{dr}{r} = \frac{\mu_0 I l}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$L = \frac{\mu_0 l}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$U_B = \frac{1}{2} L I^2 = \frac{\mu_0 l I^2}{4\pi} \ln\left(\frac{b}{a}\right)$$

ESERCIZIO 7



- ① $M = ?$
 ② $\text{f.e.m.} = ?$

①

$$\oint_{\text{SPIRA}} \vec{B} \cdot d\vec{l} = M \cdot I_1$$

$$\oint_{\text{SPIRA}} (\vec{B} \cdot d\vec{l}) = \int_{\text{SPIRA}} \vec{B} \cdot d\vec{s} = \int_{\text{SPIRA}} B_x \cdot dy = \int_c^{a+c} \frac{\mu_0 I_1}{2\pi x} \cdot l \cdot dx$$

$$\int_0^l dy \int_c^{a+c} \frac{\mu_0 I_1}{2\pi x} dx = \frac{\mu_0 I_1 \cdot l}{2\pi} \int_c^{a+c} \frac{dx}{x} = \frac{\mu_0 I_1 \cdot l}{2\pi} \ln\left(\frac{a+c}{c}\right)$$

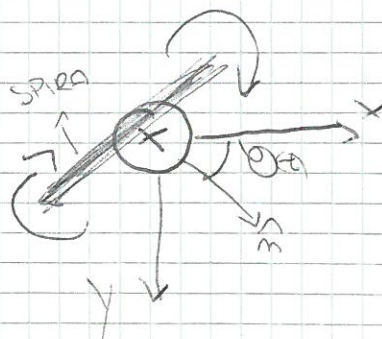
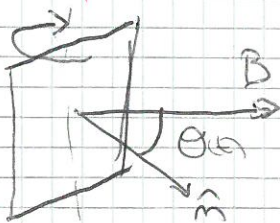
QUINDI $M = \frac{\mu_0 \cdot l}{2\pi} \ln\left(\frac{a+c}{c}\right) = 4,8 \cdot 10^{-7} \text{ Henry}$

② $I_1(t) = \alpha + \beta t$ $\beta = 10 \frac{\text{A}}{\text{s}}$

$$\oint_{\text{SPIRA}} (\vec{B} \cdot d\vec{l}) = M I_1(t)$$

$$\mathcal{E}_i = - \frac{d\oint_{\text{SPIRA}} \vec{B} \cdot d\vec{l}}{dt} = - \frac{\mu_0 \cdot l}{2\pi} \ln\left(\frac{a+c}{c}\right) \frac{dI_1(t)}{dt} = - M \frac{dI_1(t)}{dt} = - M \beta = -4,8 \cdot 10^{-6} \text{ V}$$

ESERCIZIO 8



$\omega = 60 \text{ rad/s}$

$$\theta(t) = \theta_0 + \omega t = \theta_0 + 2\pi \nu t$$

↓
LO ASSUMIAMO = 0

$$\oint_B^{\text{SPIRA}} = \int_{\text{SPIRA}} \vec{B} \cdot d\vec{S} = 0 \int_{\text{SPIRA}} B [\cos \hat{n}(t)] =$$

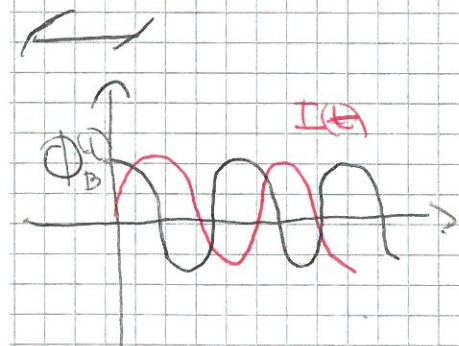
$$= \int B \cdot dS \cos(\theta(t)) = B \cos(\theta(t)) \int_{\text{SPIRA}} dS = B l^2 \cos(\theta(t))$$

$$= l^2 B \cos \omega t$$

$$\text{con } \omega = 2\pi \cdot 20$$

$$\mathcal{E}_i = - \frac{d\Phi_B}{dt} = -B l^2 \omega [-\sin(\omega t)] = \underbrace{B l^2 \omega}_{3V} \sin \omega t$$

$$I(t) = \frac{|\mathcal{E}_i|}{R} = \underbrace{B l^2 \omega}_{0.3A} \sin(\omega t) = 0.3A \sin \omega t$$



$$P = R I^2 = (0.9) W \sin^2(\omega t)$$

$$\vec{\tau} = \vec{r} \times \vec{B}$$

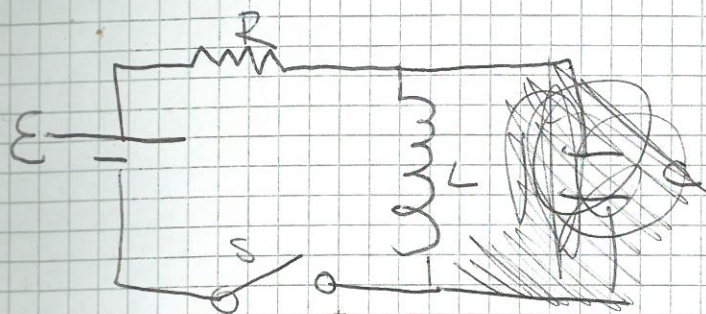
$$\vec{U} = I_{\text{indotta}} l^2 \hat{n}(t) \Rightarrow \boxed{\vec{\tau} = B I l^2 \sin(\theta(t)) \hat{z}}$$

Se voglio mantenere la spira in rotazione con 60 Hz dovrò applicare un $\vec{U}$ nel verso opposto

$$\boxed{\vec{\tau}_{\text{ext}} = -\vec{\tau}} \Rightarrow \text{ULTIMA RISPOSTA}$$

ESERCIZIO 9

$L = ?$

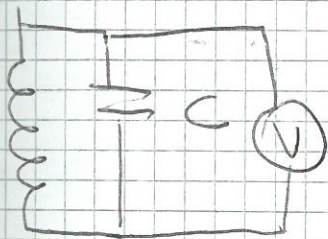


$$I(t) = I_0 (1 - e^{-\frac{t}{\tau}}) \quad \xrightarrow{t \rightarrow \infty} \quad I = I_0 = \frac{E}{R} = 0,2 \text{ A}$$

$\downarrow$ $\tau = \frac{L}{R}$ $\downarrow$
 $I_0 = \frac{E}{R}$

$$U_B^{\text{MAX}} = \frac{1}{2} L I_0^2$$

Per



Misuriamo la tensione ai capi
del condensatore $\Rightarrow \Delta V_{\text{MAX}} = 150 \text{ V}$

$$U_B + U_C = \text{cost}$$

Alla fine

$$U_B^{\text{MAX}} = \frac{1}{2} L I_0^2 = U_C^{\text{MAX}} = \frac{1}{2} C \Delta V_{\text{MAX}}^2$$

$$\Rightarrow L = \frac{C \cdot \Delta V_{\text{MAX}}^2}{I_0^2} = 0,28 \text{ Henry}$$

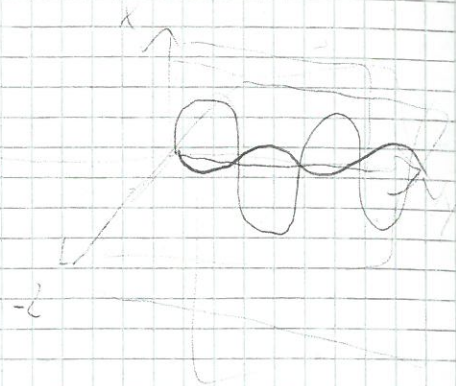
$$\omega = \frac{1}{\sqrt{LC}} = 2,7 \text{ kHz}$$

ESERCIZIO 1 (PARTE ONDE)

$$\vec{E} = \left(31 \frac{V}{m} \right) \cos \left[\left(1,8 \frac{rad}{m} \right) y + \left(5,4 \cdot 10^8 \frac{rad}{s} \right) t \right] \hat{x}$$

① DIREZIONE?

$$\vec{E} = E_0 \cos(ky + \omega t) \hat{x} \quad \vec{v} = v(-\hat{y})$$



② $\lambda = \frac{2\pi}{k} \approx 628$

③ $v = \frac{\omega}{k} \approx 36 \text{ MHz}$

④ $E_0 = 31 \frac{V}{m} \Rightarrow B_0 = E_0 = 10^{-7} \text{ T}$

⑤ $\vec{B} = B_0 \cos(ky + \omega t) \hat{z} \Rightarrow B = 10^{-7} \cos(1,8 y + 5,4 \cdot 10^8 t) \hat{z}$

⑥ $S = \frac{1}{\mu_0} \vec{E} \times \vec{B} = \frac{E_0 B_0}{\mu_0} \cos^2(ky + \omega t) \hat{y} \Rightarrow$ stesso dir e prop. all'onda

⑦ $I = \frac{1}{2} \frac{E_0 B_0}{\mu_0} \approx 1,3 \frac{W}{m^2}$

Integrare sul coseno

IMPARARE
TUTTO BENE
SUL VETTORE DI
POINTING

ESERCIZIO 2

$\cos^2 \equiv 1$
colpo

$B_0 = 18 \mu T \quad E_0 = c B_0 = 540 V/m$

$$\begin{aligned} \mu_{E+B} &= \mu_E + \mu_B = \frac{1}{2} \epsilon_0 E_0^2 + \frac{1}{2} \frac{B_0^2}{\mu_0} = \frac{1}{2} \epsilon_0 (c B_0)^2 + \frac{1}{2} \frac{B_0^2}{\mu_0} \\ &= \frac{1}{2} \frac{B_0^2}{\mu_0} + \frac{1}{2} \frac{B_0^2}{\mu_0} = \frac{B_0^2}{\mu_0} = 2,6 \frac{\mu J}{m^3} \end{aligned}$$

$S_0 = \frac{1}{\mu_0} E_0 B_0 = 773 \frac{W}{m^2}$

$S = S_0 \cos^2(kz + \omega t) = \frac{1}{2} S_0$

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$$\vec{S} = \left(-220 \frac{W}{m^2} \right) \cos^2 \left[12 \frac{rad}{m} z + 3,6 \cdot 10^9 \frac{rad}{s} t \right] \hat{k}$$

① $\vec{v} = v(-\hat{z})$

② $\lambda = 2\pi \approx 0,5 \text{ m}$

③ $f = \frac{\omega}{2\pi} = 570 \text{ MHz}$

④ $\vec{E} = E_0 \cos(kz + \omega t) \hat{y}$

$\vec{B} = B_0 \cos(kz + \omega t) \hat{x}$

$E_0 = \sqrt{S_0 \mu_0 c} = 288 \frac{V}{m}$

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$\vec{E} \times \vec{B} = \vec{v} = (-\hat{z}) \left(\frac{E_0 B_0}{\mu_0} \cos^2(kz + \omega t) \right) \Rightarrow S_0 = \frac{E_0 B_0}{\mu_0} = 2960 \frac{W}{m^2}$