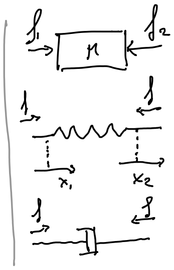
MODELLISTICA

$-\!\!\mathrm{WW}\!\!-\quad V = Ri$

$-||-\quad v = \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt \Rightarrow D_{vc} = \frac{i}{C}$

$-\mathrm{mm}-\quad v = L\,Di$

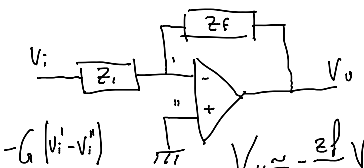


$M D^2 x = f_1 - f_2$

$f = k(x_1 - x_2)$

$f = B(v_1 - v_2)$

OPERAZ. CON BIPOLI



$-G(v_i' - v_i'')$

$G \gg 1$

$V_U \simeq -\frac{Z_f}{Z_i} V_i$

LAPLACE

$$F(s) = \int_{0^-}^{+\infty} f(t)\, e^{-st}\, dt$$

$\mathcal{L}[Df] = sF(s) - f(0^+)$

$\mathcal{L}[D^i f] = s^i F(s) - \sum_{j=0}^{i-1} s^j D^{i-1-j} f_+$

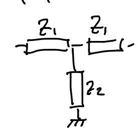
$\mathcal{L}\left[\int_0^t f(v)\,dv\right] = \frac{1}{s} F(s)$

FRATTI SEMPLICI

$F(s) = \frac{b(s)}{(s-p_1)(s-p_2)\dots} \longrightarrow \frac{k_1}{(s-p_1)} + \frac{k_2}{(s-p_2)} + \dots$

$k_{ij} = \frac{1}{(j-1)!} D^{j-1}\left[(s-p_i)^{r_i} F\right]\Big|_{s=p_i}$

TRIPOLO



$Z = Z_1 + Z_1 + \frac{Z_1 \cdot Z_1}{Z_2}$

$\mathcal{L}^{-1}\left[\frac{k}{s-p} + \frac{\bar{k}}{s-\bar{p}}\right] = 2|k| e^{\mathrm{Re}\, p t} \cos(\mathrm{Im}\, p t + \arg k)$

$\mathcal{L}^{-1}\left[\frac{1}{(s-a)^n}\right] = \frac{1}{(n-1)!} t^{n-1} e^{at}$

$\mathcal{L}\{\sin \beta x\} = \frac{\beta}{s^2 + \beta^2}$

$\mathcal{L}\{\cos \beta x\} = \frac{s}{s^2 + \beta^2}$

VALORE FINALE, INIZIALE

$\lim_{t\to+\infty} f = \lim_{s\to 0} sF \qquad \lim_{t\to 0^+} f = \lim_{s\to+\infty} sF$

TRASLAZIONE TEMPO, S

$\mathcal{L}[f(t-t_0)\cdot 1(t-t_0)] = F \cdot e^{-t_0 s}$

$\mathcal{L}[f e^{at}] = F(s-a)$

$\mathcal{L}[1(t)] = \frac{1}{s} \qquad \mathcal{L}[t\,1(t)] = \frac{1}{s^2}$

NOTEVOLI

$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}} \qquad \mathcal{L}[e^{at}] = \frac{1}{s-a}$

$\mathcal{L}[\delta] = 1 \qquad \mathcal{L}[\delta^{(1)}] = s \qquad \mathcal{L}[D^{*i} f] = s^i F - \sum_{j=0}^{i-1} s^j D^{i-1-j} f_-$

DERIVATE GENERALIZZ.

$f = g + (f_+ - f_-)1$

$D^* f = Df + (f_+ - f_-)\delta$

$D^{*n} f = D^n f + (D^{n-1} f_+ - D^{n-1} f_-)\delta + \dots + (f_+ - f_-)\delta^{(n-1)}$

$c_{-1} + c_0\delta + c_1\delta^{(1)} + \dots$ ← COND INIZIALI!

$d_{-1} + d_0\delta + d_1\delta^{(1)} + \dots$

IDENTITA' SSE

$c_0 = d_0,\ c_1 = d_1 \dots$

$\sum^n a_i D^{*i} y = \sum^m b_i D^{*i} u$

FUNZIONI TRASF.

$\sum^n a_i D^{*i} y = \sum^m b_i D^{*i} u$

risposta libera

$Y = \frac{\sum^n b_i s^i}{\sum a_i s^i} U + \frac{\sum_{i=1}^{n}\sum_{j=0}^{i-1} a_i D^{i-1-j} y_- s^j - \sum_{i=1}^{m}\sum^{i-1} b_i \dots}{\dots}$

$Y = G \cdot U$ ← risp forzata

guadagno statico $\rightarrow k = \frac{b_0}{a_0}$

polinomio caratteristico $\rightarrow \sum^n a_i s^i$

$g_s(t)$ ← risp a gradino

mod: $\rightarrow e^{pt}, t e^{pt} \dots$

VASCHY

$y_{for}(t) = \int_0^t u'(v)\, g_s(t-v)\,dv + u(0+)\, g_s(t)$

$= g_s * u$

## SISTEMI DINAMICI PRIMO ORD.

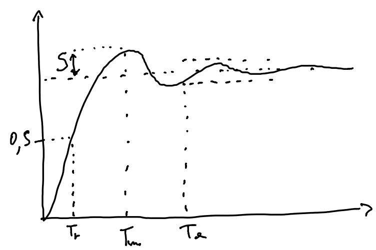
$$G(s) = \frac{1}{1+\tau s}$$

$\tau$ = cost. di tempo

**RISP. A GRADINO**

$1 - e^{-t/\tau} \quad t \geq 0$

$t = 3\tau \Rightarrow 95\%$




PER SISTEMI DI ORDINE SUP. SI APPROSSIMA CON POLI DOM.

→ COMPLESSI CONIUGATI → ORDINE 2

## SIST. DIN. SECONDO ORD.

$$G(s) = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

$\omega_n$ → pulsazione naturale

$\delta$ → coeff. smorzamento

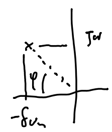
POLI: $-\delta\omega_n \pm j\omega_n\sqrt{1-\delta^2}$

MODI: $e^{-\delta\omega_n t} \sin\left(\underbrace{\omega_n\sqrt{1-\delta^2}}\, t + \varphi_1\right)$

**RISP. GRADINO**

$1 - Ae^{-\delta\omega_n t} \sin(\omega t + \varphi)$

$A = \frac{1}{\sqrt{1-\delta^2}} \qquad \varphi = \arctan \frac{\sqrt{1-\delta^2}}{\delta}$




$$S = 100 \exp\left\{-\frac{\delta\pi}{\sqrt{1-\delta^2}}\right\} \quad T_a \simeq \frac{3}{\delta\omega_n} \quad T_s \simeq \frac{1.8}{\omega_n}$$

## STABILITA'

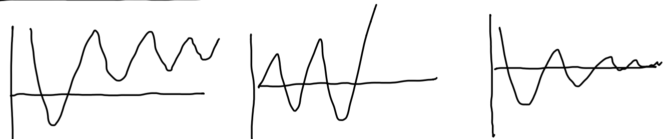
→ Stabile se nessun polo a parte reale positiva

→ Asintot. se nessun polo sull'asse Im

→ Instabile se polo parte reale Im con moltep > 1

→ BIBO → azione forzante limitata ⇒ risp. forzata limitata

BIBO ⟺ ASINT. STABILE



STABILE INSTABILE ASINT. STABILE

**EQ. CARATTERISTICA**

$a_n s^n + \dots + a_0 = 0 \longleftarrow 1 + L(s)$

**SINGOLARITA' ROUTH**

→ PRIMO ELEMENTO NULLO

$\rho$ = numero di zeri

moltiplica per $(-1)^\rho$ e traslo verso sx di $\rho$ posti

→ POI SOMMA A ULTIMA RIGA

→ TUTTI ELEMENTI NULLI

bisogna trovare polinomio ausiliario

| | |
|---|---|
| $2i$ | $\gamma_{n-2i,1} \dots$ |
| $2i-1$ | $0 \quad 0 \dots 0$ |

$\beta(s) = \gamma_{n-2i,1} s^{2i} + \gamma_{n-2i,2} s^{2i-2} + \dots \longrightarrow \beta(s) = 0$ → DERIVA → I COEFF. SOSTITUISCONO GLI ZERI NELLA TABELLA DI ROUTH

↓

PERMANENZE = RADICI PURAMENTE IMMAGINARIE

## CRITERIO ROUTH


$$\gamma_{k,j} = -\frac{\begin{vmatrix} \gamma_{k-2,1} & \gamma_{k-2,j+1} \\ \gamma_{k-1,1} & \gamma_{k-1,j+1} \end{vmatrix}}{\gamma_{k-1,1}}$$

PRIMA COLONNA

→ VARIAZIONE ⇒ $Re^+$ → INSTABILE

→ PERMANENZA ⇒ $Re^-$

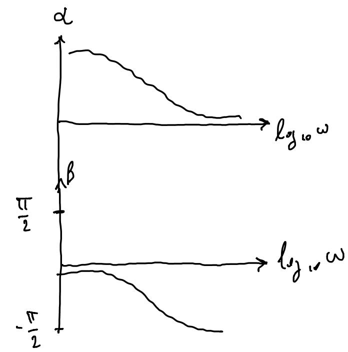
POSSIBILE MOLTIPLICARE PER k

# DIAGRAMMI BODE

$F(\omega) = G(j\omega)$

AMPIEZZE $\rightarrow$ $\alpha = \log_{10} |G(j\omega)|$

FASI $\rightarrow$ $\beta = \arg(G(j\omega))$




$\rightarrow$ SI POSSONO SOMMARE IN CASCATA I DIAGRAMMI

## FORME STANDARD

$$k_1 \frac{s^m + b_{m-1}s^{m-1} + \dots + b_0}{s^n + a_{n-1}s^{n-1} + \dots + a_0}$$

costante di trasferimento

$$k_1 \frac{(s-z_1)(s-z_2)\dots}{(s-p_1)(s-p_2)\dots}$$

## PULSAZIONE RISONANZA

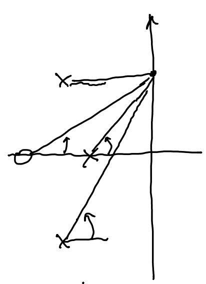
$W_R = \arg\max |G(j\omega)|$

PICCO $\rightarrow$ $M_R := |G(j\omega_R)|$

$$k \frac{(1+\tau_1' s)(1+\tau_2' s)\dots\left(1+2\delta_i'\frac{s}{\omega_{ni}'}+\frac{s^2}{\omega_{ni}'^2}\right)\dots}{s^h(1+\tau_1 s)(1+\tau_2 s)\dots}$$

$h = 0$ TIPO ZERO
$1$ TIPO UNO
$2$ TIPO DUE

# DIAGRAMMI NYQUIST

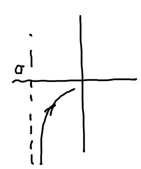


$\arg = \sum \angle z - \sum \angle p$

$k_1 > 0$

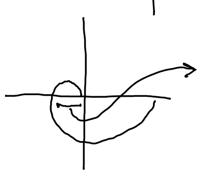
TIPO 0 $\rightarrow$ parte da punto su asse reale

TIPO 1 $\rightarrow$ asintoto



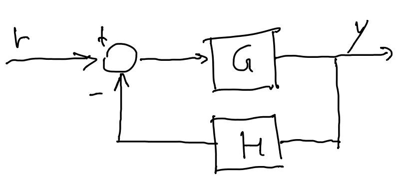

$$\sigma_a = k_1 \frac{a_1 b_1 - a_2 b_0}{a_1^2} =$$

$$= k\left[\sum \tau_i' - \sum \tau_i + \underbrace{\sum 2\frac{\delta_i'}{\omega_{ni}'} - \sum 2\frac{\delta_i}{\omega_{ni}}}_{\text{COMP. CONIUG.}}\right]$$



- passi fase $\pi$
- traccia $\omega$

# SISTEMI RETROAZIONATI

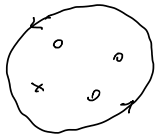



$$T_{ry} = \frac{G}{1+GH}$$

$L = GH$

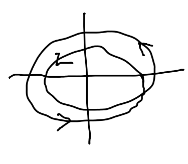
STABILE $\iff$ RADICI $1+L(s)$ PARTE $\mathbb{R}$ NEGATIVA

## TEOR. INDICE LOG.



GIRI CURVA IMMAGINE ANTI-ORARIO ATTORNO ORIGINE $= \underline{\underline{N_z - N_p}}$

$\mathbb{C}_+$ SEMIPIANO POSITIVO

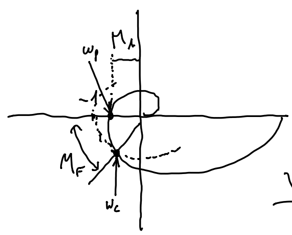


## CRITERIO NYQUIST

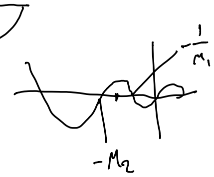
SIST ASINTOTICAM. STABILE
→ diag polare NON tocca −1 lo circonda ↺ tante volte quanti sono i poli in $\mathbb{C}_+$

CASO PARTICOLARE
→ se non ci sono poli $\in \mathbb{C}_+$ allora non deve toccare né circondare −1

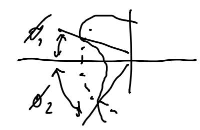
## MARGINI STABILITÁ




$$M_{A,B} \in (1,+\infty) = 20 \log M_A$$




$$M_A = \min\{M_1, M_2\}$$




$$M_F = \min\{\phi_1, \phi_2\}$$

## LUOGO RADICI

- tanti rami quanti sono i poli
- zeri attirano i poli
- mantenere simmetria asse reale
- poli si respingono tra di loro
- sempre complessi coniugati

DIRETTO $k>0$ → FA PARTE LUOGO PUNTO CHE A DX HA NUMERO DISPARI POLI E ZERI

$k<0$ → PARI

## INTERSEZIONE ASINTOTI

$$\sigma_a = \frac{\sum p_i - \sum z_i}{n-m} \quad \leftarrow \text{ASCISSA}$$

$$\vartheta_{a,\nu} = \frac{(2\nu+1)\pi}{n-m} \quad \nu = 0,1\dots n-m-1 \quad \leftarrow \text{ANGOLI}$$

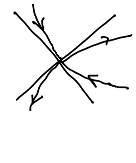
## ANGOLO PARTENZA

$$\varphi = \underset{\substack{\uparrow \\ k>0}}{\pi} + \sum_{z_0}^{z_n} \angle(p_i - z_j) - \sum_{\substack{i \neq j \\ p_0}}^{p_n} \angle(p_i - p_j)$$

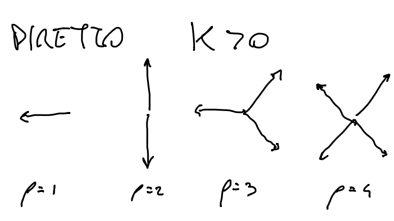
altrimenti non c'è

PER ANGOLI DI ARRIVO SI SOSTITUISCE $z_i$ A $p_i$

RADICE DEL LUOGO CON MOLTEPLICITÁ H
→ PUNTO IN COMUNE A H RAMI
→ H RAMI ENTRANTI, H USCENTI
→ DIVIDONO LO SPAZIO IN SETTORI $\pi/H$

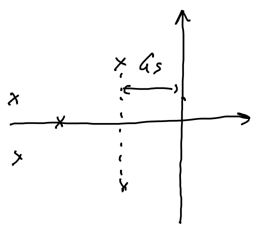


$\rho$ NUMERO ASINTOTI

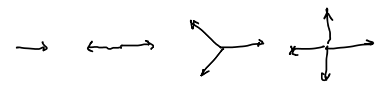


## GRADO STABILITÀ

$$G_s = -\max\{Re\,p_1, Re\,p_2 \dots\}$$




INVERSO $K<0$

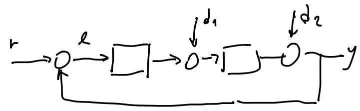


SE ∃ COPPIA DOMINANTE

$$T_a \approx \frac{3}{G_s}$$

# PROGETTO SISTEMA RETROAZIONE

Sistema asintoticamente e internamente stabile quando tutte FDT fra ingressi e uscite sono asint. stabili




$\{r, d_1, d_2\}$

$\{e, m, y\}$

SIST. STABILE SSE
- RADICI PARTE REALE NEGATIVA
- CANCELLAZ. POLO-ZERO IN $\mathbb{C}^-$

$$S = T_{re} = \frac{1}{1+L} \qquad T_{ry} = 1 - S$$

SENSITIVITÀ

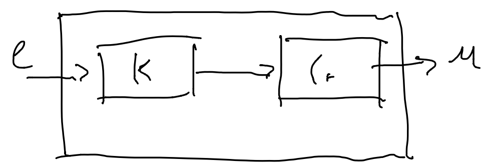
## ORDINI CONTROLLORI

0: $C(s) = k$

1: $C(s) = \dfrac{b_1 s + b_0}{s + a_0}$

2: $C(s) = \dfrac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0}$

definiti mediante parametrizzazione funzione di trasferimento




**INTEGRATRICE**

$$C_r = \frac{1}{1+\tau s} \quad \tau > 0$$

**DERIVATRICE**

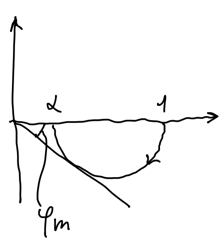
$$C_r = \frac{\tau s}{1+\tau s} \quad \tau > 0$$

→ AZIONE COMPENSATRICE

→ FORMULE INVERSIONE

**RITARDATRICE**

$$C_r = \frac{1+\alpha\tau s}{1+\tau s} \quad \alpha \in (0,1) \quad \tau > 0$$

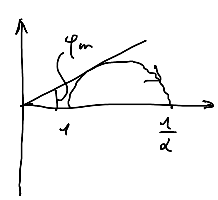



$$\varphi_m = -\arcsin\frac{1-\alpha}{1+\alpha}$$

$$\omega_m = \frac{1}{\sqrt{\alpha}\cdot\tau}$$

**ANTICIPATRICE**

$$C_r = \frac{1+\tau s}{1+\alpha\tau s} \quad \alpha \in (0,1) \quad \tau > 0$$




$$\varphi_m = \arcsin\frac{1-\alpha}{1+\alpha}$$

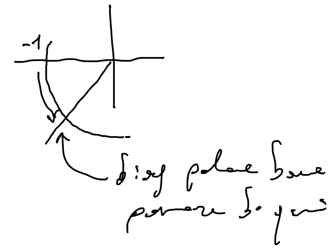
→ esempio corretto su 2ª [illegible]

## FORMULE INVERSIONE

$$Me^{j\varphi} = \frac{1+jx}{1+j\alpha x} \qquad M = \frac{\sqrt{1+(\tau\omega)^2}}{\sqrt{1+(\alpha\tau\omega)^2}}$$

$$\begin{cases} \alpha = \dfrac{M\cos\varphi - 1}{M(M-\cos\varphi)} \\ x = \dfrac{M-\cos\varphi}{\sin\varphi} = \underline{\underline{\tau\omega}} \end{cases} \qquad \varphi = \mathrm{atg}(\tau\omega) - \mathrm{atg}(\alpha\tau\omega)$$

MARGINE FASE




RETE ANTICIPATRICE $M_F$

→ SCEGLIERE $\omega_0$ AFFINCHÉ

$$\varphi_0 := M_F - \arg L(j\omega_0) - \pi$$
$$\cos\varphi_0 > |L(j\omega_0)|$$

$$M := \frac{1}{|L(j\omega_0)|} \qquad \varphi = \varphi_0$$

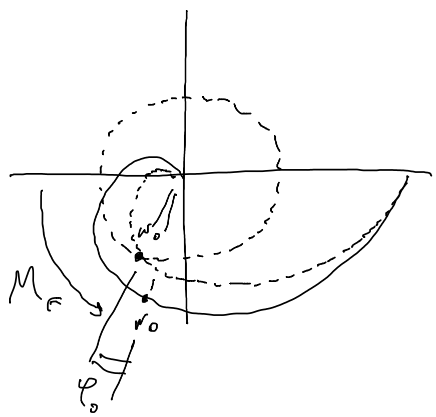
RETE RITARDATRICE $M_F$

→ SCEGLIERE $\omega_0$ AFFINCHÉ

$$\varphi_0 := \arg L(j\omega) + \pi - M_F$$
$$\cos\varphi_0 > \frac{1}{|L(j\omega_0)|}$$

$$M := |L(j\omega_0)| \qquad \varphi = \varphi_0$$

→ $\tau$, $\alpha$

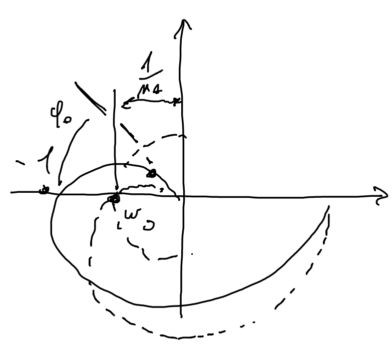



RETE ANTICIPATRICE $M_A$

→ $\omega_0$ AFFINCHÉ

$$\varphi_0 = -\arg L(j\omega_0) - \pi$$
$$\cos\varphi_0 > M_A |L(j\omega_0)|$$

$$M := \frac{1}{M_A |L(j\omega_0)|} \qquad \varphi = \varphi_0$$



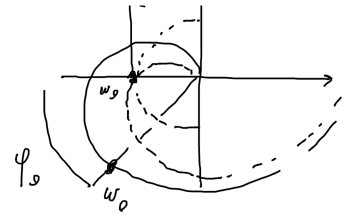

RETE RITARDATRICE $M_A$

→ $\omega_0$ AFFINCHÉ

$\varphi_0 = \arg L(j\omega_0) + \pi$

$\cos\varphi_0 > \dfrac{1}{M_A |L(j\omega_0)|}$

$M := M_A |L(j\omega_0)| \quad \varphi = \varphi_0$




→ OMISSIS RETI T, AR, DIOFANTEA ECC...

# CONTROLLO DIGITALE

## CAMPIONAMENTO SHANNON

SPETTRO LIMITATO $0 - \omega_s$ RICOSTRUIBILE DA $\tilde{f}(k) = f(kT)$ $\Leftrightarrow$ PULSAZ. CAMPIONAMENTO $\Omega = 2\pi/T \geq 2\omega_s$

## TRASF. Z

$$\mathcal{Z}[x(k)] = \sum_{k=0}^{\infty} x(k) z^{-k}$$

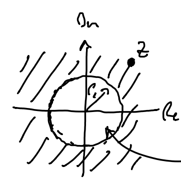
$z$ var complessa

$$X(z) = x(0) + x(1)z^{-1} + \dots$$

La serie è convergente nella regione ROC

$\{|\sum^{\infty} x(k) z^{-k}| < +\infty\} \rightarrow |z| > \rho_c$

$\rho_c$ raggio assoluta convergenza




se z è più dentro diverge a $+\infty$

SEGNALE RITARDATO N PASSI

ANTICIPO → $+$ ... $-$ ... $(i)\, z^{n-i}$

$$\mathcal{Z}[x(k-n)] = z^{-n}\mathcal{Z}[x(k)] + \sum_{k=0}^{n-1} x(k-n) z^{-k}$$

→ COND INIZIALI NULLE

$$\mathcal{Z}[x(k-n)] = z^{-n} X(z)$$

$$\mathcal{Z}[a^k x(k)] = X\left(\frac{z}{a}\right)$$

VALORE INIZIALE

$$x(0) = \lim_{z \to +\infty} X$$

VALORE FINALE

$$\lim x(+\infty) = \lim_{z \to 1} (z-1) X$$

CONVOLUZIONE

DERIVATA

PARABOLA

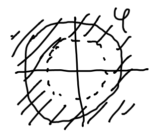
$(x * y)(k) = \sum_{i=-\infty}^{+\infty} x(k-i)\, y(i)$ $\mathcal{Z}[x * y] = X * Y$

$-z \dfrac{dX}{dz} = \mathcal{Z}[k\, x(k)]$ $\mathcal{Z}[k^2] = \dfrac{z(z+1)}{(z-1)^3}$

DOVE ANALITICA

ANTITRASF. Z

- RESIDUI
- SVILUPPO FRATTI



$x(k) = \dfrac{1}{2\pi j} \oint_{\gamma} X(z)\, z^{k-1}\, dz$

$\Rightarrow x(k) = \sum_i \mathrm{Res}\left\{X(z) z^{k-1}, p_i\right\}$ SINGOLARITÀ DISTINTE

METODO RESIDUI

$\mathcal{Z}\left[\binom{k}{h-1} a^{k-(h-1)} \cdot 1(k)\right] = \dfrac{z}{(z-a)^h}$

$\mathcal{Z}\left[\dfrac{1}{(z-a)^h}\right] = \underbrace{\dfrac{(k-1)(k-2)\cdots(k-h+1)}{(h-1)!}}_{\binom{h-1}{h-1}} a^{k-h}\, 1(k-1)$

... CASI PART ...

## SISTEMI TEMPO DISCRETO

eq. alle differenze

$f(y(k), y(k-1), \dots, y(k-n)) = g(u(k-n+m), u(\dots) \dots u(k-n))$

$\Rightarrow a_n y(k) + a_{n-1} y(k-1) + \dots + a_0 y(k-n) = b_m u(k-n+m) \dots$

ANALISI SISTEMA
- dalle eq. alle differenze
- assegnati ingressi e cond. iniziali $\begin{cases} y(-1) \\ y(-2) \\ \vdots \\ u(-n+m) \end{cases}$
- determinare uscita

TRASF Z DI SEGNALE CAMPIONATO

$y(k) = y_{FOR} + y_{LIB}$

$y_{FOR} = \mathcal{Z}^{-1}\left[\underbrace{\dfrac{b(z)}{a(z)}}_{\text{F TRASF}} U(z)\right]$ $y_{LIB} = \mathcal{Z}^{-1}\left[\dfrac{\overbrace{c(z)}^{\text{GARBUGLIO}}}{a(z)}\right]$

DA TEOR CONVOLUZIONE

$y_{FOR}(k) = \sum_k h(k-i)\, u(i)$

## FUNZIONE TRASF CON DAC

$$\tilde{u} \to \overbrace{\boxed{A/D} \xrightarrow{u} \boxed{P} \to \boxed{D/A}}^{\Sigma_d} \to \tilde{y}(k)$$

$$\mathcal{Z}[f(t), T] \triangleq \mathcal{Z}[f(kT)] = \mathcal{Z}[F(s), T]$$

## RISPOSTA IMPULSO

$$P_d(z) = \mathcal{Z}\left[(1 - e^{-Ts}) \frac{P(s)}{s}, T\right]$$

$$\Sigma_d \quad = \frac{z-1}{z} \mathcal{Z}\left[\frac{P(s)}{s}, T\right]$$

## STABILITÁ SIS TEMPO DISC.

-> sistema $\Sigma_d$ con fdt $H(z)$

- STABILE $\longleftrightarrow$ poli appartengono a cerchio unitario, molteplicità 1
- ASINT. STABILE $\longleftrightarrow$ tutti poli interno cerchio unitario
- SEMP. STABILE $\longleftrightarrow$ almeno un polo su circonf. unitaria, molteplicità 1
- INSTABILE $\longleftrightarrow$ esterni circ. unitaria

**BIBO STABILE**

per ogni azione forzante limitata risposta limitata

$$\iff \sum_{k=0}^{+\infty} |h(k)| < +\infty$$

$\iff$ ASINT. STABILE

## DETERMINAZ. STABILITÁ ASINTOTICA CRITERIO JURY

-> determinare radici con modulo < 1

-> tutte disuguaglianze soddisfatte

$$\begin{cases} a(1) > 0 \\ (-1)^n a(-1) > 0 \\ |a_0| < a_n \\ |b_0| > |b_{n-1}| \\ |c_0| > |c_{n-2}| \\ \vdots \\ |r_0| > |r_2| \end{cases}$$

$n$ = grado max

per $n = 2$ -> DIVENTANO SUFFICIENTI

-> tabella Jury associata a polinomio

$$\begin{array}{c|cccc} & z^0 & & & z^n \\ \hline 1 & a_0 & \cdots & & a_n \\ 2 & a_n & \cdots & & a_0 \\ 3 & & & & \\ 4 & & & & \\ \vdots & & & & \\ & s_0 & s_1 & \cdots & \\ & s_3 & \cdots & & \end{array}$$

$\leftarrow b_k = \begin{vmatrix} a_0 & a_{n-k} \\ a_n & a_k \end{vmatrix}$

$\leftarrow c_n$

$r_0 = \begin{vmatrix} s_0 & s_3 \\ s_3 & s_0 \end{vmatrix} \quad r_1 = \begin{vmatrix} s_0 & 2 \\ 3 & s_1 \end{vmatrix} \quad r_2 = \begin{matrix} 0 & 1 \\ 3 & 2 \end{matrix}$

-> OMESSI EULERO AVANTI, INDIETRO, TUSTIN