

Appendice A

Tabella delle Z-trasformate

$f(t)$	$\mathcal{L}[s]$	Trasformata $\mathcal{Z}$	Trasformata $\mathcal{Z}$ modificata
$h(t)$	$\frac{1}{s}$	$\frac{z}{z-1}$	$\frac{1}{z-1}$
t	$\frac{1}{s^2}$	$\frac{Tz}{(z-1)^2}$	$\frac{mT}{z-1} + \frac{T}{(z-1)^2}$
t^2	$\frac{1}{s^3}$	$\frac{T^2 z(z+1)}{2(z-1)^3}$	$\frac{T^2}{2} \left[\frac{m^2}{z-1} + \frac{2m+1}{(z-1)^2} + \frac{2}{(z-1)^3} \right]$
t^{k-1}	$\frac{(k-1)!}{s^k}$	$\lim_{a \rightarrow 0} (-1)^{k-1} \frac{\partial^{k-1}}{\partial a^{k-1}} \left[\frac{z}{z-e^{-aT}} \right]$	$\lim_{a \rightarrow 0} (-1)^{k-1} \frac{\partial^{k-1}}{\partial a^{k-1}} \left[\frac{e^{-amT}}{z-e^{-aT}} \right]$
e^{-at}	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$	$\frac{e^{-amT}}{z-e^{-aT}}$
te^{-at}	$\frac{1}{(s+a)^2}$	$\frac{Tze^{-aT}}{(z-e^{-aT})^2}$	$\frac{Tze^{-amT}[e^{-aT} + m(z-e^{-aT})]}{(z-e^{-aT})^2}$

$e^{-at} \sin bt$	$\frac{b}{(s+a)^2 + b^2}$	$\frac{ze^{-aT} \sin bT}{z^2 - 2ze^{-aT} \cos bT + e^{-2aT}}$	$\frac{e^{-amT} [z \sin bmT + e^{-aT} \sin(1-m)bT]}{z^2 - 2ze^{-aT} \cos bT + e^{-2aT}}$
$e^{-at} \cos bt$	$\frac{s+a}{(s+a)^2 + b^2}$	$\frac{z^2 - ze^{-aT} \cos bT}{z^2 - 2ze^{-aT} \cos bT + e^{-2aT}}$	$\frac{e^{-amT} [z \cos bmT + e^{-aT} \sin(1-m)bT]}{z^2 - 2ze^{-aT} \cos bT + e^{-2aT}}$
$1 - e^{-at} (\cos bt + \frac{a}{b} \sin bt)$	$\frac{a^2 + b^2}{s[(s+a)^2 + b^2]}$	$\frac{(Az+B)z}{(z-1)(z^2 - 2ze^{-aT} \cos bT + e^{-2aT})}$ $A = 1 - e^{-aT} (\cos bT + \frac{a}{b} \sin bT)$ $B = e^{-2aT} + e^{-aT} (\frac{a}{b} \sin bT - \cos bT)$	$\frac{1}{z-1} - \frac{e^{-amT} [z \cos bmT + e^{-aT} \sin(1-m)bT]}{z^2 - 2ze^{-aT} \cos bT + e^{-2aT}} + \frac{s \{ e^{-amT} [z \sin bmT - e^{-aT} \sin(1-m)bT] \}}{z^2 - 2ze^{-aT} \cos bT + e^{-2aT}}$
$\frac{1}{ab} + \frac{e^{-at}}{a(a-b)} + \frac{e^{-bt}}{b(b-a)}$	$\frac{1}{s(s+a)(s+b)}$	$\frac{(Az+B)z}{(z-e^{-aT})(z-e^{-bT})(z-1)}$	$A = \frac{b(1-e^{-aT}) - a(1-e^{-bT})}{ab(b-a)}$ $B = \frac{ae^{-aT}(1-e^{-bT}) - be^{-bT}(1-e^{-aT})}{ab(b-a)}$

$t^k e^{-at}$	$\frac{k!}{(s+a)^{k+1}}$	$(-1)^k \frac{\partial^k}{\partial a^k} \left[\frac{z}{z-e^{-aT}} \right]$	$(-1)^k \frac{\partial^k}{\partial a^k} \left[\frac{e^{-amT}}{z-e^{-aT}} \right]$
$1 - e^{-at}$	$\frac{a}{s(s+a)}$	$\frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})}$	$\frac{1}{z-1} - \frac{e^{-amT}}{z-e^{-aT}}$
$t - \frac{1-e^{-at}}{a}$	$\frac{a}{s^2(s+a)}$	$\frac{z[(aT-1) + e^{-aT}]z + (1-e^{-aT} - aT e^{-aT})}{a(z-1)^2(z-e^{-aT})}$	$\frac{T}{(z-1)^2} + \frac{amT-1}{a(z-1)} + \frac{e^{-amT}}{a(z-e^{-aT})}$
$1 - (1+at)e^{-at}$	$\frac{a^2}{s(s+a)^2}$	$\frac{z}{z-1} - \frac{z}{z-e^{-aT}} - \frac{aT e^{-aT} z}{(z-e^{-aT})^2}$	$\frac{1}{z-1} - \left[\frac{1+amT}{z-e^{-aT}} + \frac{aT e^{-aT}}{(z-e^{-aT})^2} \right] e^{-amT}$
$e^{-at} - e^{-bt}$	$\frac{b-a}{(s+a)(s+b)}$	$\frac{(e^{-aT} - e^{-bT})z}{(z-e^{-aT})(z-e^{-bT})}$	$\frac{e^{-amT}}{z-e^{-aT}} - \frac{e^{-bmT}}{z-e^{-bT}}$
$\sin at$	$\frac{a}{s^2 + a^2}$	$\frac{z \sin aT}{z^2 - 2z \cos aT + 1}$	$\frac{z \sin amT + \sin(1-m)aT}{z^2 - 2z \cos aT + 1}$
$\cos at$	$\frac{s}{s^2 + a^2}$	$\frac{z(z - \cos aT)}{z^2 - 2z \cos aT + 1}$	$\frac{z \cos amT - \cos(1-m)aT}{z^2 - 2z \cos aT + 1}$