Soluzione di alcuni esercizi da Oppenheim e Willsky p.47

Es. 2.5

(f)

$$z = je^{j\pi\frac{11}{4}} = e^{j\frac{\pi}{2}}e^{j\pi\frac{11}{4}} = e^{j\pi\frac{13}{4}} = e^{-j\pi\frac{3}{4}} = -\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}$$
$$Re\{z\} = -\sqrt{2}/2$$
$$Im\{z\} = -\sqrt{2}/2$$

(h)

Il numero complesso z che ha modulo $|z| = \sqrt{2}$ e argomento $arg(z) = -\frac{\pi}{4}$ ha parte reale e immaginaria come segue:

$$Re\{z\} = |z|cos(arg(z)) = \sqrt{2}\ cos(-\pi/4) = \sqrt{2}\ \frac{\sqrt{2}}{2} = 1$$
$$Im\{z\} = |z|sen(arg(z)) = \sqrt{2}\ sen(-\pi/4) = \sqrt{2}\ \frac{-\sqrt{2}}{2} = -1$$

E quindi: $z = \sqrt{2}\ e^{-j\frac{\pi}{4}} = 1 - j$

(i)

Usando il risultato di (h):

$$
\begin{aligned}
w = (1-j)^9 &= \left(\sqrt{2}\, e^{-j\frac{\pi}{4}}\right)^9 = \left(\sqrt{2}\right)^9 \; e^{-j\frac{9}{4}\pi} \\
&= 16\,\sqrt{2}\, e^{-j\frac{\pi}{4}} = 16(1-j)
\end{aligned}
$$

E quindi:

$$
\begin{aligned}
Re\{w\} &= 16 \\
Im\{w\} &= -16
\end{aligned}
$$

Es. 2.6

(g)

$$
\begin{aligned}
z = (\sqrt{3} + j^3)(1-j) &= (\sqrt{3} - j)(1-j) \\
&= 2\, e^{-j\frac{\pi}{6}} \; \sqrt{2}\, e^{-j\frac{\pi}{4}} = 2\sqrt{2}\, e^{-j\pi\left(\frac{1}{6}+\frac{1}{4}\right)} = 2\,\sqrt{2}\, e^{-j\pi\frac{5}{12}}
\end{aligned}
$$

E quindi:

$$
\begin{aligned}
|z| &= 2\,\sqrt{2} \\
arg(z) &= -\pi\frac{5}{12}
\end{aligned}
$$

(j)

$$
z = j(1+j)\, e^{j\frac{\pi}{6}} = \left(e^{j\frac{\pi}{2}}\; \sqrt{2}\, e^{j\frac{\pi}{4}}\right) e^{j\frac{\pi}{6}} = \sqrt{2}\, e^{j\pi\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}\right)} = \sqrt{2}\, e^{j\pi\frac{11}{12}}
$$

E quindi:

$$
\begin{aligned}
|z| &= \sqrt{2} \\
arg(z) &= \pi\frac{11}{12}
\end{aligned}
$$

(k)

$$
\begin{aligned}
z &= (\sqrt{3}+j)\,2\,\sqrt{2}\,e^{-j\frac{\pi}{4}} \\
&= 2\,e^{j\frac{\pi}{6}}\;2\,\sqrt{2}\,e^{-j\frac{\pi}{4}} \\
&= 4\,\sqrt{2}\,e^{j\pi\left(\frac{1}{6}-\frac{1}{4}\right)} \\
&= 4\,\sqrt{2}\,e^{-j\frac{\pi}{12}}
\end{aligned}
$$

E quindi:

$$
\begin{aligned}
|z| &= 4\sqrt{2} \\
arg(z) &= -\frac{\pi}{12}
\end{aligned}
$$

Es. 2.7

(a)

Sia $z = x + jy$, allora:

$$(e^z)^* = (e^{x+jy})^*$$

Il numero complesso fra parentesi ha modulo e^x e argomento y, quindi:

$$(e^{x+jy})^* = e^x\,e^{-jy} = e^{x-jy} = e^{z^*}$$

(d)

Siano $z_1 = r_1 e^{j\theta_1}$ e $z_1 = r_2 e^{j\theta_2}$, quindi $r_1 = |z_1|$ e $r_2 = |z_2|$.

$$|z_1\,z_2| = \left|r_1\,e^{j\theta_1}\;r_2\,e^{j\theta_2}\right| = \left|r_1\,r_2\,e^{j(\theta_1+\theta_2)}\right| = r_1\,r_2 = |z_1||z_2|$$