Teoria dei Segnali – Filtri tempo-discreti

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Contenuto

Digital Filters

Filters are used for band limiting signals (*anti-aliasing*), for band splitting in multiplexing/demultiplexing, for pulse forming in digital modulators, for decimation and interpolation (e.g., in $\Sigma\Delta$ A/D and D/A data converters), for equalization, ...

Filters may be:

- linear
- non-linear
- adaptive

Linear Digital Filters

Linear filters are **linear time-invariant** (LTI) systems
$\longrightarrow$ they are completely described by their impulse response $h(t)$ (or $h[k]$)

For sampled-data systems (including digital systems), filtering may be described in the Z-domain:

$$H(z) = \mathcal{Z}\{h[k]\} = \sum_{k=-\infty}^{+\infty} h[k] z^{-k}$$

Linear filters may have:

- finite impulse response (FIR)
- infinite impulse response (IIR)

FIR Filters

The system function is

$$H(z) = \sum_{k=0}^{N-1} h[k] z^{-k}$$

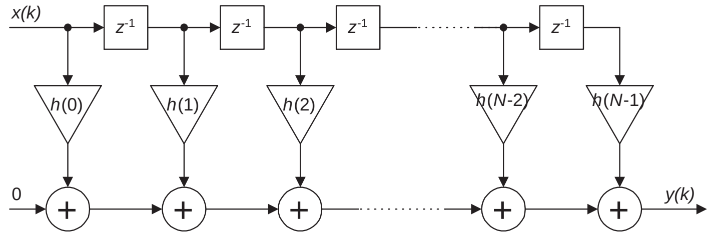
The output is

$$y[m] = \sum_{k=0}^{N-1} h[k] x[m-k]$$

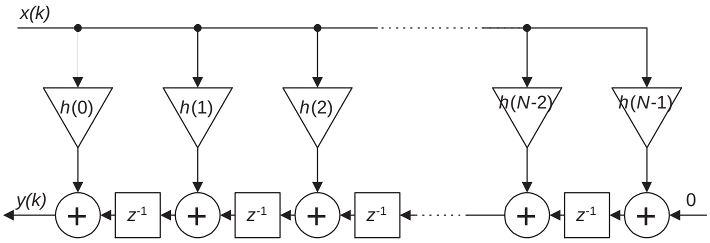
By applying an impulse $\delta[k]$ to the input, the output goes to zero after N samples.

An FIR filter is also called a **moving average** (MA) filter.

Signal Flow Graphs of an FIR Filter



Direct form I



Direct form II

Linear Phase FIR filters

In DSP and communications, phase must be proportional to the frequency (*linear phase*):

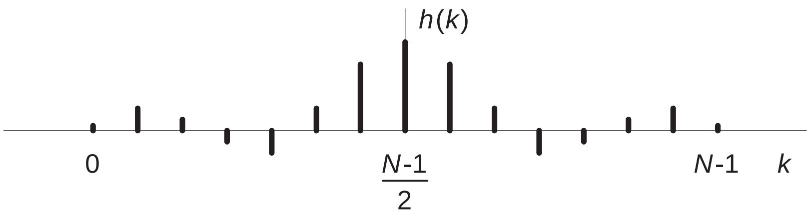
$$\varphi(f) = \angle H(f) \propto f$$

Linear phase corresponds to a pure delay in time domain: all frequency components are delayed by the same time amount.

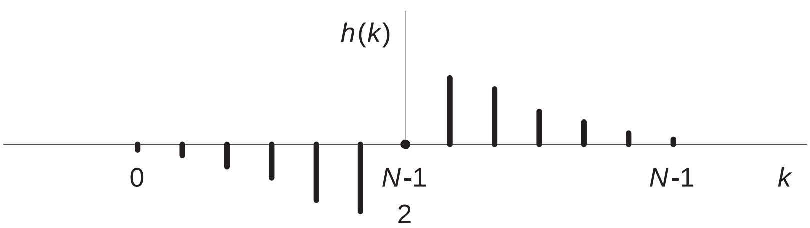
A linear phase FIR filter has **symmetrical impulse response** (either even-symmetrical or odd-symmetrical):

$$h[k] = h[N-1-k] \qquad \text{even-symmetric}$$
$$h[k] = -h[N-1-k] \qquad \text{odd-symmetric}$$

Impulse Response of Linear Phase Filters

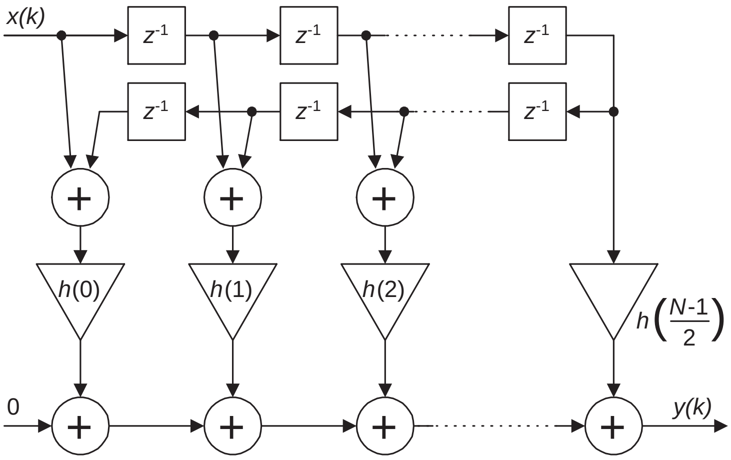


Even symmetry



Odd symmetry

SFG of a Symmetrical FIR Filter



Direct form I

This solution saves about one half of the multiplications.

IIR Filters

The system function is

$$H(z) = \frac{B(z)}{A(z)} = \frac{\sum_{k=0}^{N-1} b[k]z^{-k}}{1 - \sum_{j=1}^{M} a[j]z^{-j}}$$

The output is

$$y[m] = \sum_{k=0}^{N-1} b[k]x[m-k] + \sum_{j=1}^{M} a[j]y[m-j]$$

By applying an impulse to the input, the output does not go to zero after a finite number of samples.

AR IIR Filters

When $B(z) = 1$, the output sample $y[m]$ is a linear regression of previous output values. Such a filter is called an **autoregressive** (AR) filter.

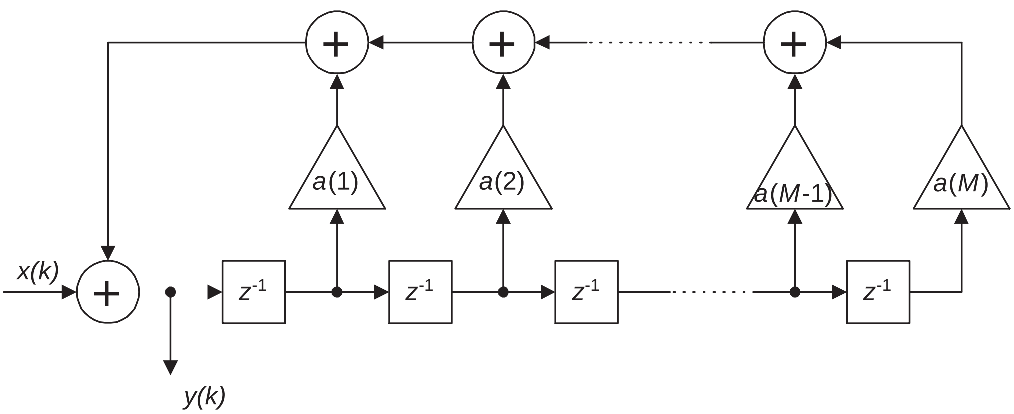
The system function is

$$H(z) = \frac{1}{A(z)} = \frac{1}{1 - \sum_{j=1}^{M} a[j] z^{-j}}$$

The output is

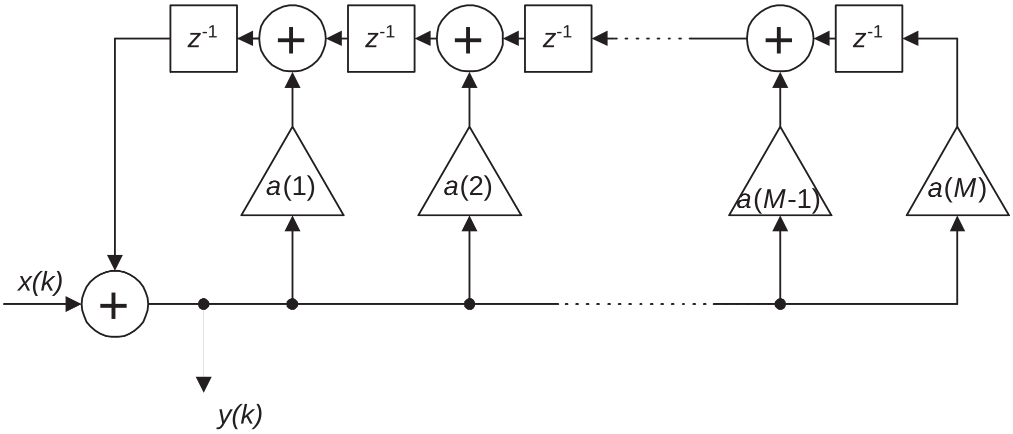
$$y[m] = x[m] + \sum_{j=1}^{M} a[j] y[m-j]$$

SFG of an AR IIR Filter (I)



Direct form I

SFG of an AR IIR Filter (II)



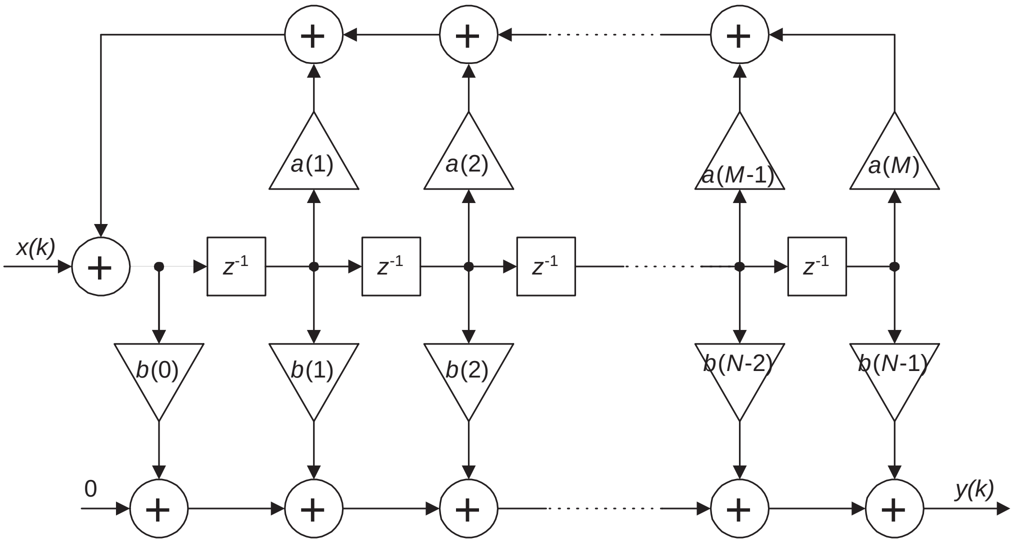
Direct form II

ARMA IIR Filters

When $B(z) \neq 1$, the filter is an **autoregressive, moving average** (ARMA) filter.

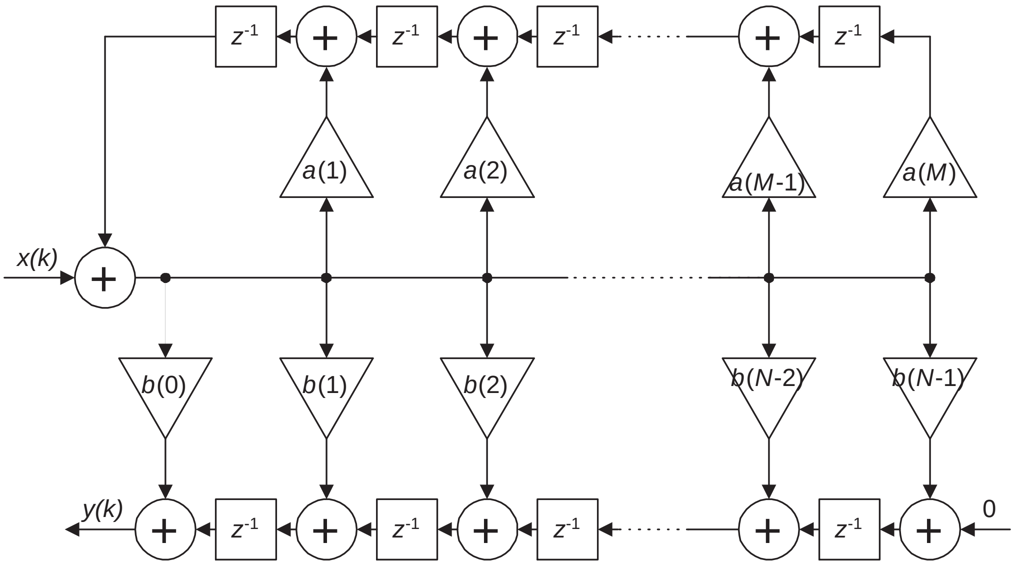
The signal flow graph can be obtained by combining the SFGs of the AR part and of the MA part.

SFG of an ARMA IIR Filter (I)



Direct form I

SFG of an ARMA IIR Filter (II)



Direct form II (non-canonical)

Note: it is NOT required to have $M = N - 1$.

FIR vs IIR Filters

FIR			IIR
more coefficients	☹	☺	**less coefficients**
smooth cut-off in transition band	☹	☺	**sharp cut-off in transition band**
linear phase	☺	☹	no linear phase (only approximation possible)
always stable	☺	☹	limit cycles may occur due to finite arithmetics $\longrightarrow$ oscillations!